A
KEY
TO THE
Modern Sliding-Rule.

CONTAINING

The DESCRIPTION, and EXLPANATION of the various PURPOSES, of that valuable INSTRUMENT, as now used by his MAJESTY's OFFICERS of CUSTOMS, EXCISE, &c.

AND ALSO,

Of TWO IMPROVED SLIDING-RULES, for speedily and accurately GAUGING and MEASURING SOLIDS and SUPERFICIES at ONE OPERATION.

Not to be performed by any other INSTRUMENT yet constructed.

TOGETHER WITH

The ADVANTAGES of a NEW INSTRUMENT of SLIDING SINES and TANGENTS in PLANE and SPHERICAL TRIGONOMETRY.

By the Rev. Wm. FLOWER, A. M.
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Projector of the said Instruments.

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MDCCLXVIII.

TO THE

HONOURABLE COMMISSIONERS

OF HIS

MAJESTY'S ROYAL REVENUES

OF

CUSTOMS AND EXCISE;

THIS TREATISE OF THE

MODERN SLIDING-RULE,

IS MOST HUMBLY INSCRIBED,

BY THEIR OBEDIENT SERVANT,

W. FLOWER.

THE

PREFACE.

SO many are the treatises extant on the use of the *Sliding-Rule*, that I am conscious the publication of the following sheets will need some apology: I shall therefore observe, that among all the books on this subject, which I have hitherto perused, (and those are not a few) there is not one, I know of, which hath done it that justice its *utility* deserves; neither of them having given any manner of *rule*, whereby the *answer* to any question proposed to be solved thereby, may be truly and justly *ascertained*; which I humbly conceive to be no small neglect: for though it may be possible, and in some cases very easy, to know the *value* of the answer; nevertheless I am certain, that in many cases there will be found some difficulty in ascertaining the true *number*, or value, of *places* therein,

particularly, when one or more of the *factors* concerned consists of more than one or two *integral*, or intire *fractional* places; and it is further to be observed, that this difficulty will still be greater, when the *line* D or E is concerned in the proportion.

I have conversed with several persons who are esteemed *masters* of the *Sliding-Rule*, who I have found, could give no other proof of the *truth* of their *solutions* of those problems they were daily concerned with, (viz. why the number of *places* in their answers, should not be *more* or *fewer* than what they had hitherto assigned;) than that their *reason* suggested to them, it could not be *more*; and that therefore they concluded it could not be *fewer*. But how satisfactory this *answer* may appear to an ingenious and inquisitive mind, I leave to the *judgment* of the reader: and I dare say, if one of these *gentlemen* should happen to be attacked with a *proposition* wherein some of the *data* should exceed, or fall short in *number* of places, those he hath usually met with in practice, his *Golden Rule* will be found quite insufficient for producing a *speedy* or *accurate* answer.

Another *motive* to my publishing the following sheets, was the hopes I had of thereby removing that *indifference* or rather *dislike*, which I have observed to have appeared in many *artizans* to the *use* of this most admirable *instrument*, and which must arise, either from their apprehensions of some great *difficulty* in the right management thereof,

or

or of ſome *uncertainty* or *inaccuracy* in its performances; which *prejudices* cannot, I conceive, ſo ſoon or effectually be removed, as by ſhewing, that all the moſt *common*, *uſeful* and *neceſſary*, as well as *curious* and *delightful* problems, concerning the *calculation* of weights, meaſures, &c. of almoſt all kinds, where the anſwer doth not exceed *three* integral places, may be ſolved by the inſtrument, not only as *truly* and *juſtly* (in regard to common uſe) but much more *eaſily* and *expeditiouſly* than by the moſt dextrous *pen*.

To ſay nothing of the manifeſt advantages of that *inſtrument* which more particularly gave *riſe* to the following ſheets, nor of the peculiar manner which the *uſe* of the *Modern Sliding-Rule* in general is therein treated of, I ſhall here inſert a *tranſcript* of part of Mr. *Leadbetter*'s preface to his laſt edition of the *Royal Gauger*, which ſhews the great importance of the *uſe* of this moſt admirable *inſtrument*, in the following words, viz.

" As ſome writers have attempted to perſuade
" the public, that *tables* ready calculated, are far
" more exact and ready in practical *gauging* than
" the *Sliding-Rule*, it may not be here amiſs to
" obſerve, that if *tables* happen to be falſe printed,
" as we frequently find moſt tables are, the offi-
" cer muſt act at *random*, not knowing whether
" he is right or wrong: whereas by the *Sliding-*
" *Rule*, it is impoſſible he ſhould ever err; for
" the uſe of that *inſtrument* being but once well

"understood,(*which, by the directions given in "the following treatise, it very easily may) the "officer, with the greatest *dispatch* and *certainty* "may, on all occasions, come to the *exactness* of "the tenth part of an *unit*, which is as near as "is ever required in practice in the *excise*; and "therefore the *author* is persuaded, that those "who have taken the most *pains* to decry the "*Sliding-Rule*, are truly ignorant of its *excellency* "and *use*."

Having given my *reasons* for appearing in print, I cannot, without doing great violence to *custom*, omit saying something of the *work itself*.

But what I have to advance on this head, must fall very short of the usual *encomiums* on the like occasion; and it is this, I am conscious, the work itself will be found in *faults* very abundant; yet as it is solely and purely designed for the *instruction* and *use* of the *unlearned*, I am not without hopes of its meeting with a favourable *reception* by my *candid* readers.

What I have to add shall be addressed to those of my *readers*, whom I have so inadvertantly denominated *unlearned*; but for so rude an *appellation*, I shall first beg pardon; tho' no other was the object of the *allusion* than the *instrument*.

To those of my readers, then, I shall observe, that in order to their reaping that *benefit* from their *perusal* of the following sheets, which the *author* in penning them intended they should, it

* This parenthesis is more justly applicable to the following Treatise, than to Mr. Leadbetter's.

will

will be neceſſary that they read and apply them to uſe, with the ſame *regularity*, and *order*, in which they will find them written.

The reaſon hereof will be found very obvious; becauſe not only the *chapters* themſelves, but alſo each *rule* and *example* in each *chapter*, will be frequently found dependant on ſome preceding *rule* or *chapter*.

I ſhall now point out the other moſt material *particulars*, which are neceſſary to be known, and remembered by all who intend to be *maſters* of this moſt *uſeful* inſtrument.

1. Then, it is neceſſary they ſhould have a right underſtanding of the *prime* and *collateral radius*; alſo of the *ſum*, as well as *difference*, of places in any two or more given numbers: of *ſimple*, *duplicate*, and *triplicate proportions*; and to obſerve, in what ſort of *meaſure*, *weight*, *dimenſions*, &c. the queſtion to be ſolved is propoſed, and the *anſwer* required.

2. Another eſſential *requiſite*, in order to the rightly ſolving any *problem* by the inſtrument, is the operator's knowledge of the *value*, in number of places, of the proper *factor*, *diviſor*, or *gauge-point*, with which he is at any time more immediately concerned.

And for this reaſon each of the ſaid *agents* on the inſtrument is marked in ſuch manner, as that the *value* of each may be readily known at ſight, whereon the certainty of aſcertaining a true anſwer in all caſes entirely depends.

3. It

3. It is moreover neceſſary, that the reader ſhould know when the *principal agent* in any propoſition be a *factor*, *diviſor*, or *gauge-point*; alſo, by what particular *lines* of the inſtrument the ſaid *propoſition* is to be ſolved; that he may not only the more readily find the ſaid *agent* thereon; but alſo, that he may the better and more readily be able to aſcertain the true *value* of any anſwer. But this, and alſo how to find the particular *factor*, *diviſor*, &c. required, on the inſtrument in any propoſition, are fully ſhewn in their proper places; I mention them here only, as they are ſome of the moſt neceſſary *requiſites* to be known and remembered by the *operator*.

I have but one obſervation more to make, and that is in regard to *calculations* on the inſtrument, which, though it may be looked upon as very frivolous; yet I am perſuaded the *practitioner* will find it of very great *uſe*. It is this;

When any number concerned in any *proportion* is ſuch as is not found exactly expreſſed by any diviſion of the *inſtrument*, but muſt be ſuppoſed to be repreſented by ſome *imaginary* point between ſome two *diviſions*; the *point* repreſenting ſuch *number* muſt be eſtimated by the eye; which may be effected moſt nearly in the following manner.

You are not to ſuppoſe the *diſtance* between any two next diviſions to be divided into 10 *equal*, but into ſo many *unequal* parts; and the *diſtance* of ſuch ſuppoſed 10 parts from each other, muſt be imagined to *decreaſe* from the *left* hand

hand toward the *right*, on the *direct* lines; but the contrary on the *inverted* lines; in the same proportion as do the *tenths* and *centisms* of the same prime from each other.

Thus, suppose it be required to place the *prime* 3 on B, to the *intermediate* point 695 on A.

I say the *point* or prime 3 on B, must not be placed exactly against the middle between the two last divisions or *tenths*, of the prime 6 on A, but somewhat more to the *right* hand of the said middle, viz. nearer the prime 7; because all the *intermediates* become *nearer* to each other as they proceed from their respective *primes* towards the *right*, consequently the *distance* of the point which represents 695, must be further from that which represents 690, than it is from that *point* which represents 700, viz. the prime 7; and so on of any other.

I must here inform my *reader*, that the following sheets which relate to the sliding *sines* and *tangents*, were not designed to have appeared at this time; but by the advice and desire of a worthy friend and *ingenious mathematician*, I have herewith published them, in hopes thereby, that this *treatise* will become more acceptable and useful to the *public*.

The method wherein I have treated this subject is the same with that of the other lines; both which, I am persuaded, will be found to be quite *new*, and I hope, *satisfactory*.

My

My reader, perhaps, may think I have not been so *full* and *explicite* as I ought to have been in this part of my *treatise*, particularly in that which relates to *oblique* spheric triangles: but as my *business* is not to treat more particularly of *trigonometry*, but of the use of the instrument in *trigonometrical* calculations in general, I hope what I have said therein will be sufficient to that purpose.

It may not perhaps be disagreeable to some of my readers, if I here shew, how each *line* is laid down on each *instrument*.

I. *For the Lines on the* Officers *and* Artificers *Instrument.*

Draw a right line FG, just *twice* the length of your intended

H
F————————|————————G

radius A, B, or C, and divide it into *two* equal parts in the point H; so will the line F, H, and H, G, be each equal in *length* to the *radius* A, B, or C; and the *whole line* FG, equal to *radius* D.

Parallel to FG, and *equal* thereto, draw *three* lines to represent the *radii* or *lines* A, D, and E.

Divide the line FG, and GH, or suppose them to be divided each into 10.000 *equal* parts; so will the whole line FG be divided into 20.000 *equal* parts.

From

From the *point* F, *perpendicularly* on the lines A, D, and E, draw a right line; and the points, on each line, whereat the ſaid *perpendicular* falls, will be the *prime* point 1 of each reſpective *radius*. Thus:

1. For *radii* A, B, and C.

Seek in *Sherwin*'s Tables of Logarithms for the natural number

2000	and find its logarithm, viz.	3010, &c.	and from this *point* on FG, draw a right line *perpendicularly* on the line A, and it will be the *point* whereon is to be placed the *prime*	2
3000		4771		3
4000		6020		4
&c.		&c.		&c.

2. For *radius* D.

Double the above *logarithm*, and it will become

6020, &c.	which *point* found on FG, and transferred as above, on the *line* D, will point out thereon the *prime* - - -	2
9542		3
2041		4
&c.		&c.

3. For *radius* E.

Divide the laſt *logarithm* by 3, and the quotient will be

2006, &c.	which being transferred as above on the *line* E, will give thereon the *prime* point -	2
3180		3
6804		4
&c.		&c.

II. *Of the* Lines *of* Sines *and* Tangents.

Having, by the above method, laid down 3 *radii* of the line of numbers A, B, and C, in a right

right line, and *parallel* thereto drawn two *right lines* to represent the lines of *sines* and *tangents*. Then,

1. For *sines*.

Seek in the tables the *natural* sine of

1	degree, and it will be found to be	1747, &c.	Find this *point* on radius A and transfer it on the line of *sines*, and it will shew thereon the *point* - - - -	1
2		3489		2
3		5233		3
&c.		&c.		&c.

2. For *tangents*.

Seek in the tables the *natural* tangent of

1	degrees, which will be found to be	1745, &c.	which *point* on A, being transferred to the line of *tangents* as above, will shew thereon the *prime* point - - - -	1
2		3492		2
3		5240		3
&c.		&c.		&c.

In like manner may the rest of the *primes* and *intermediates* of each line be transferred.

III. *Of the Line of* versed Sines.

Having laid down a line of *numbers* consisting of three *radii*, which call A, B, and C, draw a *right* line parallel and equal thereto, to represent the line of V, *sines*; and thereon right against the *prime* 1 of *radius* C, place the *point* or *radius* 90 degrees; and against *prime* 2 of the same *radius* C, place a *cypher*, to denote the first point or *beginning* of the line of *versed sines*. Then,

1. For

1. For *versed sines* greater than the *radius*.

Seek in the *tables* the natural V. sine of

80	degrees, which will be found to be	8263, &c.	and from this point of *radius* B, draw a *perpendicular* on the line V. *sines*, and it will cut the said line in the *point*	80	deg. which number by its supplement, viz.	100
70		6579		70		110
60		5000		60		120
50		3572		50		130
&c.		&c.		&c.		&c.

2. For *versed sines* less than *radius*.

Find the *complement* of the said *versed sine* to *radius* 10, and thereto prefix the *radius* or prime 1, and it will become

1.173, &c.	and from this point of *radius* C, draw a *right line* perpendicularly on the line of *versed sines*, and it will point out thereon the *place* of its *prime*, viz. - - - -	80
1.3420		70
1.5000		60
1.6427		50
&c.		&c.

In the same manner may the rest of the *primes* and *intermediates* be laid down.

N. B. All these *lines* may be laid down independant of the *lines*, A B, and C, by the *tables* of artificial *sines*, *tangents* and *V. sines*.

As it would swell this treatise beyond its intended price, to have introduced it, as usual, by a prefatory discourse on *fractional* arithmetic: besides there are so many *books* extant on that subject, which may be had at very reasonable rates; and few *persons*, I presume, whose inclinations lead them to any knowledge of *this kind*, but have one or the other of them; and as I have, in

their proper places, shewn how to convert any *vulgar fraction* into its equivalent *decimal* and *è contra*, I hope I shall be excused in not complying with *custom* in this particular.

I cannot in justice to the memory of the worthy and ingenious *inventor* of the lines which constitute these most valuable *instruments*, conclude without acquainting my reader, that the *learned mathematician*,

Mr. EDMUND GUNTER,

Was born in 1580; bred at *Christ-Church* Oxon; succeeded *Edward Brerewood*, professor of astronomy at *Gresham-College*, in November 1613; became famous for his *tables* of artificial *sines* and *tangents*, which were printed in latin octavo 1620; and for his elaborate conclusions on the *sector* and *forestaff*: he died in the year 1623.

The first edition of his works is printed with additions by *William Leybourn*, who tells us, what *plagiarisms* have been made upon him.

For the above *anecdote*, I must own myself obliged to my late worthy and ingenious friend *William Oldyss*, Esq; Norroy King at Arms.

London,

London, Dec. 16, 1766.

AT the *author's* requeſt, I have carefully examined two *Sliding-Rules*, contrived by the Reverend Mr. *William Flower*; the *one* adapted to the uſe of his Majeſty's *Cuſtoms* and *Exciſe*, (if it ſhall ſo pleaſe the *Honourable Commiſſioners*) the *other* to the purpoſes of *Artificers* in general; and I think them far preferable to any *Sliding-Rule* I have yet ſeen, for the *variety*, *facility*, and *accuracy* of their *operations*, and for their *portable* ſize. I have alſo peruſed his *manuſcript* containing the *conſtruction* and *uſes* of the ſaid *Sliding-Rules*, which I find drawn up with great *judgment* and *perſpicuity*; ſo as to be not only a compleat *Key* (as it is called) to theſe particular *Sliding-Rules*; but likewiſe applicable, in all reſpects, to *any other*.

J. BEVIS.

HAVING particularly examined *two new Sliding-Rules*, constructed by the Reverend Mr. *William Flower*; do give it as my opinion, that they are well *contrived*; are concise and extensive in *operation*; of a very convenient size; and well deserve the *notice* of all such as are concerned in the business of *gauging*, and measuring, *superficies* and *solids*; particularly the Officers of his Majesty's *Customs*, *Excise*, &c. to whose *use* they are more peculiarly adapted than any I have before seen.

The *treatise* of their use is *succinct* and *clear*, and fully explains the true *nature* and *rationale* of these, and the several other kinds of *Sliding-Rules* now in use; so as to leave no *ambiguity* or *doubt* in the learner, what *value* to affix to the *answer*; the same being here *exactly* ascertained by a *new*, *easy* and *general rule*.

Royal Academy, Woolwich,
March 10, 1767.

J. L. COWLEY,

HAVING

HAVING been desired by the Reverend Mr. *Flower*, to examine his new invented *Sliding-Rules*, together with his *treatise* upon that subject; in justice to his ingenuity, I recommend the said *instruments*, as the most *useful* of their kind I have ever seen, both for *accuracy* and *expedition*, in *gauging*, *mensuration*, &c. they solving at *one operation*, such *problems* as require *two* or *more* by the *common* Sliding-Rules.

In the *treatise* abovementioned, he has explained their *uses* in such a *clear* and *judicious* manner, that no person, who reads it with any degree of *attention*, can be under the least *difficulty* with regard to their *application*.

Royal Academy, Portsmouth,
August 1767.

G. WITCHELL.

BY

BY the desire of the Reverend Mr. *Flower*, I have examined two new invented *Sliding-Rules* of his contrivance; *one* adapted to the *use* of his Majesty's Officers of *Customs* and *Excise*; the *other* of *Artificers* in general: also his manuscript *treatise* containing the *description* and *uses* thereof: and my opinion is, that the said *instruments* are more particularly *well* adapted to their intended *uses*, than any I have hitherto seen; and that the said *treatise* is very worthy the *attention* of all persons concerned in *gauging*, or *mensuration* of any kind; it being not only a *Key* to the abovesaid *instruments*; but also to all *Sliding-Rules* now in use.

London,
Oct. 16, 1767.

SAM. CLARK.
Teacher of Mathematics.

CON-

CONTENTS.

PART I.

DEscription of the Instruments, with their Parts; of the Radii, with their Primes and Intermediates, and their Estimation, Numeration, &c. together with Tables of 194 fixed Factors, Divisors, and Gauge-points; with the Use of the Line A, *in gauging and measuring of* Areas *and* Superficies.

PART II.

Of the inverted Line I, *in the Rule of Three Inverse, and Compound Multiplication and Division, with the Use of the Divisors thereon, in gauging and measuring of Areas, Superficies, and Solids, at* one Operation.

PART III.

Description of the Line D, *with its Use in extracting the Roots of* Squares, *and in the* Proportions *of like Areas and Superficies: Also, in* gauging *and* measuring *of Areas, Superficies and Solids.*

PART IV.

Description of the Line E, *with its Use in extracting of the Roots of* Cubes, *and in finding the* Solidities *of the five* Platonicks *or regular Bodies: Also in finding their* Weight *in Stone, Lead, Iron, Box, and Marble; and in* Proportions *of similar and like* Solids.

PART V.

Containing the Construction *and Use of Tables of* natural *Sines and Tangents: Also, the Manner of working* Proportions *by the* sliding *Sines and Tangents, and of their Use in* plane *Trigonometry.*

CHAP. VI.

PART VI.

Containing the Use of the sliding Sines *and* Tangents *in* spheric *Trigonometry.*

INTRODUCTION.

APPENDIX.

Description and Use of four different Instruments whereon the inverted *Lines are peculiarly and respectively adapted to the Use of* Timber-merchants, Carpenters *and* Saywers, Shipwrights, Bricklayers *and* Glaziers, *in measuring of Superficies and Solids at* one Operation; *whereby the Time and Trouble of rectifying the Instrument is saved.*

DESCRIPTION.

ERRATA.

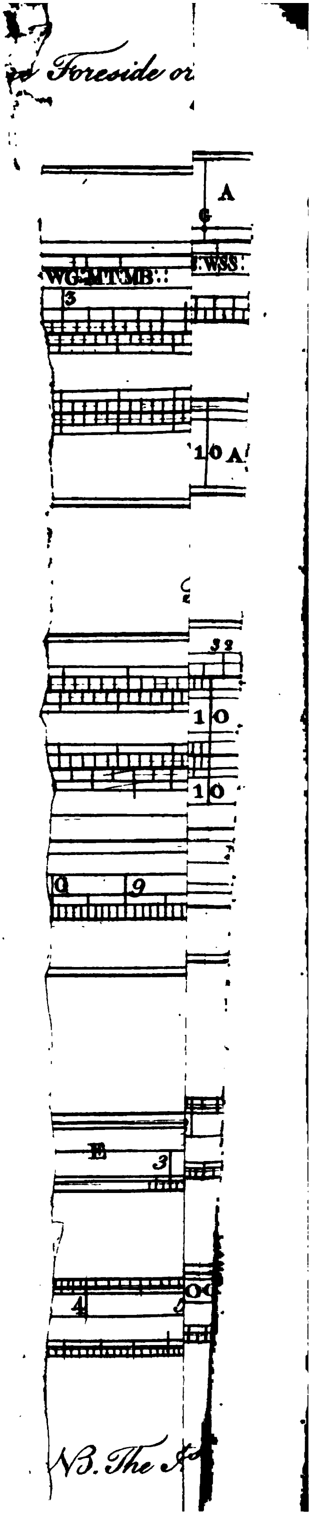
Page 3. line 2. read *measures*; p. 4. l. 19. and p. 5. l. 1. read *lesser*; p. 21. l. 7. read *proportionals*; p. 22. l. 4. for *is* read *are*; p. 23. l. 6. dele *and C*; p. 26. l. 26. read *consist*; p. 30. table III. No. 26. for character *MF* read *MT*; p. 74. l. 8. and 12. read *proportionals*; p. 76. l. 19. read *division*; p. 83. l. 25. read *or brass*; p. 100. l. 8. read *primes*; p. 110. l. 3. read *radii*; p. 116. l. 8. dele *thus*; p. 121. l. 8. dele *point*; p. 123. l. 30. dele last *is*; p. 131. l. 12. read *the platonicks*; p. 134. l. l. 9. read *dodecaedron*; p. 143. l. 3. read *integral*; p. 149. l. 16. read *than the*; p. 150. l. 8. read *proportions*; p. 151. l. 9. and 11. read *parallelopepid*; p. 153. l. 8. read *of proportions*; p. 161. l. 16. read *divided*; p. 165. l. 15. read *edges of the*; l. 19. read *the lower edges*; p. 175. l. 6. read *of tangents*.

A TABLE of ſuch Symbols or Characters, as are made Uſe of in this Treatiſe, with their Significations.

+	is the ſign of	Addition, or more.
—	- - - - -	Subtraction, or leſs.
×	- - - -	Multiplication, or by, or into.
÷	- - - - -	Diviſion, or by.
=	- - - -	Equality, or is equal to.
:	- - - - -	Proportion, or as, or to.
::	- - - -	Ditto, or ſo is.
∴	- - - - -	Therefore.

THESE Inſtruments are accurately made by Mr. JOHN BENNET, Mathematical Inſtrument Maker to their *Royal Highneſſes* the Duke of *Glouceſter* and Duke of *Cumberland*, in Crown-court, St. Anne's, Soho, *London*; where any perſon may have any particular *Factor*, *Diviſor*, or *Gauge-point*, put on any Line of either *Inſtrument*, by directing a Line (Poſt paid) to the ſaid Mr. *Bennet*, ſpecifying the *Number* of ſuch *Factor*, &c. as they ſtand regiſtered in the *Tables*.

A KEY


Foreside or
A
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10
10
9
E
3
4
N.B. The

A

KEY

TO THE

MODERN SLIDING RULE.

PART I.

Description of the Instruments, with their Parts; of the Radii, with their Primes and Intermediates; and their Estimation, Numeration, &c. together with Tables of 194 *fixed Factors, Divisors, and Gauge Points: With the Use of the Line* A *in gauging and measuring* Areas *and* Superficies.

CHAP. I.

Description of the Instruments, with their Parts.

SECT. I. *Of the Officer's Instrument.*

THIS instrument consisteth of eight parts, *viz.* a nine-inch rule and seven sliding rods. It may be made of 12 if required.

Of the Rule.

1. On the lower edge of the foreside, viz. one of the broader planes of the rule, is put one radius of

Gunter's

Gunter's double line of numbers, called and marked, the line or radius A, and is numbered with the primes 1, 2, 3, 4, &c. up to 10, as usual.

2. On the upper edge of the same side or plane, is put a line without primes, having on it several divisors properly placed and charactered. This I call the line or radius upper A.

3. On the upper edge of the opposite plane to this, viz. the backside of the instrument, is put the single line or radius D, in a broken and doubled manner, as usual.

4. On each edge, or narrower plane of the instrument is put a line of segments for ullaging of casks: that for casks lying is marked SL, that for casks standing, SS.

Of the Slides.

1. Abutting against upper A is a slide, on whose lower edge are put parts of two radii of Gunter's double line of numbers; but in an inverted order: that on the right-hand is called and marked the radius I; that on the left I 2; on its upper edge are several divisors properly placed and charactered.

2. On the back side of the instrument under the line D, are two slides, each about half the length of the instrument, and are parts of the abovesaid inverted line I, either of which is to be used with its correspondent part I, or I 2 aforesaid; on their upper edges are also put several divisors.

These

These lines are to be used with A, B and C, in superficial and solid measurement, as will be taught below.

3. On one side of two of the other slides are put two radii of numbers, called and marked B and C, each like unto A; and are both together to be used therewith, in superficial measure, with the line I as abovesaid; and with D in superficial and solid measure, also in proportions of like areas and superficies.

4. On the backsides of these slides is put the triple line E, in a broken and doubled manner. It is to be used with the line D in the measuring the five Platonicks or regular bodies, and in finding the weights of the said bodies in diverse metals, woods and stone; also in the proportion of like solids.

5. The other two slides are the same with the slides B and C, and are to be used together with the lines of segments in ullaging of casks.

SECT. II. *Of the Artificer's Instrument.*

The lines here are exactly the same with the above, excepting only the lines of segments; instead whereof may be put lines of sines and tangents, or some useful tables.

SECT. III. *Of the Radius, with its Primes and Intermediates.*

Of the Radius.

A radius is that part or portion of any line, which is intercepted between the prime or figure 1 incluſive, and the next prime 1 above it, excluſive: viz. the next 1 towards the left hand on the inverted line; but towards the right on all others. That is, the prime 1 is the firſt point of every radius.

Of the Primes and Intermediates.

Primes are the figures 1, 2, 3, 4, &c. up to 9, which are on each radius of the inſtruments. They are ſo called, becauſe they repreſent the firſt figure of every number.

Intermediates are the ſtrokes of diviſion which are between every two primes, and do repreſent the ſecond and third figures, &c. in every number. They are of two ſorts, viz. greater and leſs.

1. The greater intermediates are uſually called tenths, whereof there are 9 between each two primes: one or other of which doth always repreſent the ſecond figure in any number, if it be not a cypher.

Thus:

If the ſecond figure be	1, 2, 3, 4, &c.	it will be expreſſed by the	firſt, ſecond, third, fourth, &c.	tenth of its proper prime.

2. The

2, The less intermediates are usually called centisms; whereof there are always 9 supposed to be between each two tenths; one or other whereof doth always represent the third figure in any number, if it be not a cypher.

Thus,

If the third figure be	1,	it will be expressed by the	first,	centism of its proper tenth.
	2,		second,	
	3,		third,	
	4, &c.		fourth, &c.	

N. B. If the second figure be a cypher, the third figure will be represented by one or other of the centisms between its proper prime and first tenth.

N. A. All the centisms are not put on between each two tenths, the distance of most of them not admitting thereof: but it is to be observed,

1. That between the primes 1 and 2 of the line D, they are put on; so that the point representing the third figure of any number, whose prime is 1, may be exactly found thereon.

2. Between the primes 2 and 3 of the same line D, also between the primes 1 and 2 of the rest of the lines of numbers, there are but four divisions between each 2 tenths, every division representing two centisms.

Hence,

If the third figure be	2,	it will be expressed by the	first,	division or centism, of its proper tenth.
	4,		second,	
	6,		third,	
	8, &c.		fourth, &c.	

Note. If the third figure be 1, 3, 5, 7, or 9, its point will be found between some two or other

of the above points. Between which two will easily be discovered.

3. Between each 2 tenths of the rest of the primes on D, and also of the rest of the primes of all the other lines up to the prime 5, there is but one division, which division represents 5 centisms, viz. 5 in the third place of any number.

If the third figure be greater or less than 5, its point must be estimated by the eye.

4. Between each 2 tenths of the rest of the primes, viz. from 5 upwards, there are no centisms: so that the point representing the third figure in any number whose prime is 5 or greater, must be estimated by the eye, by supposing 1, 4, or 9 divisions between each 2 tenths.

N. B. If you suppose 9 divisions or strokes between each two centisms of the prime 1, on the line D, such divisions are called Millenisms, and will represent the fourth figure in any number whose prime is 1, if it be not a cypher. Hence

If the second and third figures in any number be cyphers, its point will be found between the prime 1, and the first Millenism.

SECT. IV. *Estimation of Primes and Intermediates.*

The primes and intermediates are all arbitrary, and may represent units, tens, hundreds, or thousands of units, &c. or they may represent a tenth, hundredth, thousandth, or ten thousandth part of unity, or of any thing.

Thus,

Thus, the prime 5, on any radius, may represent 5, 50, 500, or 5000, &c. or it may represent .5, .05, .005, or .0005, &c.

Alſo, the firſt tenth of the prime 3, may repreſent 3.1, 31, 310, or 3100, &c. or .31, .031, .0031, or .00031, &c.

Again, the centiſm or diviſion between the ſecond and third tenth of the prime 3, may repreſent 3.25, 32.5, 325, or 3250, &c. or .325, .0325, .00325, or .000325, &c.

So the ſecond centiſm, between the 5th and and 6th tenth of the prime 2 on the line D, may repreſent 2.54, 25.4, 254. or 2540, &c. or .254, .0254, .00254, or .000254, &c.

And the ſecond centiſm of the prime 1 of the line D, viz. the ſecond diviſion or ſtroke between prime 1 and the firſt tenth, may repreſent 1.02, 10.2, 102, or 1020, &c. or .102, .0102, .00102, or .000102, &c.

Hence,

Obſerve, all integral or whole numbers having never ſo many cyphers annexed to but one and the ſame ſignificant figure; alſo all fractions having never ſo many cyphers prefixed to the ſame ſignificant figure, are repreſented at the ſame point:

Thus the integers	7. 70. 700. 7000.	alſo the fractions	.7 .07 .007 .0007	are all repreſented at the ſame point, viz. the prime 7. on any radius.

2. Obſerve alſo, all numbers conſiſting of the ſame or like figures, are found at one and the ſame point.

Thus, the integral and mixed numbers,	1.895 18.95 189.5 1895.	and the fractions	.1895 .01895 .001895 .0001895	on D, are all repreſented

at the ſame point AG, of the officer's inſtrument.

Alſo,

The integral and mixed numbers,	1.354 13.54 135.4 1354.	and the fractions	.1354 .01354 .001354 .0001354	on D of the artificer's inſtrument

ment are all repreſented at the point ⊖d;

CHAP. II.

Of the Imaginary and Collateral Radii.

1. PLACE the ſlides B at the left hand, and C at the right, between the lines A and I, or inverted line, ſo that the intermediate 95 on B, may join the intermediate 95 on C: then will the two ſlides B and C become one continued line, having on them two like and equal radii to the radius A.

2. Move the ſlides together, till the prime 1 at the beginning or left hand of B, ſtands right againſt the prime 1 at the beginning or left hand of A: then will each prime and intermediate of

A,

A, ſtand right againſt its like prime or intermediate on B.

That is, the prime 2 on A will ſtand againſt the prime 2 on B; and the prime 3 on A, againſt the prime 3 on B: ſo the intermediate 25 on A will ſtand againſt the intermediate 25 on B, &c.

Now, theſe two radii (A and B) thus ſtanding in a collateral poſition to each other, may be termed collateral radii or collaterals; and ſo the primes and intermediates thereon may be called collateral primes and intermediates.

3. Now you are to imagine another like and equal radius to A, B, or C, running upwards to the right hand from A, and abutting againſt the radius C, as A doth againſt B; each prime and intermediate of the one ſtanding right againſt its like prime and intermediate of the other, which I therefore call the collateral radius of C.

And thus may you conceive as many like and equal radii as you pleaſe, running from the radius A towards the right, with an equal number of like and equal radii proceeding in a direct line the ſame way from the radius B; each prime and intermediate of the former, at the ſame time ſtanding right againſt its like on the latter.

N. B. This poſition of the ſlides I call the direct poſition of B.

4. Move the ſlides together to the left, till the prime 1 on C ſtands right againſt the prime 1 on A; then will each prime and intermediate of the

one

one radius, stand right against its like prime and intermediate of the other.

In this position you may imagine a like and equal radius also to A, B, or C, running down from A to the left, and abutting against its collateral B; the primes and intermediates thereof standing right against their like on B.

And thus may you imagine an infinite number of radii ascending and descending from the radius A, and abutting against an infinite number of like radii, the prime and intermediate of the one, standing right against their like primes and intermediates of the other, on their respective collateral radii.

N. B. This position of the slides I call the direct position of C.

N. A. The like is to be observed with respect to the inverted lines.

All other positions of the slides are called oblique.

CHAP.

CHAP. III.

Of the Motion, Appropriation, and Translation of Primes and Intermediates, with the Determination of the Radii.

SECT. I. *Of the Motion of Primes and Intermediates.*

1. PLACE B direct; then from what hath been ſaid, as each prime and intermediate on A ſtands againſt its like on B, ſo doth each prime and intermediate on every ſuppoſed radius both above and below A, ſtand againſt its like on its reſpective imaginary collateral radius.

2. Move the ſlides together to the left, till 3 on B ſtands right againſt 1 on A: now, every radius both above and below B being ſuppoſed to be equally moved the ſame way, it is obvious, that the poſition of the primes and intermediates on every radius both above and below A, is the very ſame in regard to their reſpective collaterals, as thoſe on A are to thoſe on its collateral B.

That is, the prime 2, on each radius, both above and below A, doth now ſtand againſt the prime 6 of its collateral, as the prime 2 on A doth againſt the prime 6 of its collateral B.

And at the ſame time it is obvious, that the prime 5 on each ſuppoſed radius above and below A, muſt now be imagined to ſtand againſt the

intermediate 15 on the next radius above its collateral; as the prime 5 on A doth againſt 15 on C, the next radius above its collateral B.

Hence it follows, that all the primes and intermediates above the prime 3 on C, and alſo on every radius above C, are repreſented by the like primes and intermediates on B; and that all the primes and intermediates below the prime 3 on B, and on every radius below B, are repreſented by their like primes and intermediates on C.

Hence alſo it follows, that in every oblique poſition of the ſlides B, C, all thoſe primes and intermediates of the radius C, which ſtand above the radius A, are repreſented by the like primes and intermediates on the radius B; and all thoſe of the radius B, which ſtand below the radius A, are repreſented by the like primes and intermediates on the radius C, and conſequently, that every ſuppoſed radius both above and below the radii A, B, and C, are repreſented by A, B, and C.

SECT. II. *Of the Appropriation of Primes and intermediates, and Determination of the Radii.*

The primes and intermediates on every radius are, as hath been obſerved, all arbitrary, and may ſignify or repreſent any numbers at pleaſure: but it is to be obſerved, that as ſoon as you have appropriated any prime or intermediate, on any radius, to expreſs or repreſent any number then all the primes,

primes, and consequently their intermediates on that radius are said to be appropriated; and do then represent numbers consisting of an equal number of places with that number to which such prime or intermediate was first appropriated.

N. B. The primes and intermediates on each radius do naturally represent numbers, consisting of one place more than those on the next radius below it, or to the left.

Hence, as soon as any prime or intermediate, on any radius is appropriated, all the primes and intermediates on every radius both above and below it, are also appropriated; and the radii said to be determined, because we then know what particular number each prime and intermediate on every radius doth represent.

Example. Let the prime 3 on B represent 3 units; then is the radius B said to be appropriated to numbers consisting of one integral place; and consequently all the supposed radii both above and below are now said to be determined.

For now the primes 1, 2, 3, 4, &c. on B, do represent 1, 2, 3, 4, &c. units, and the like primes on C, do represent 10, 20, 30, 40, &c. and on the next radius above C, they represent 100, 200, 300, 400, &c.

At the same time, the primes 1, 2, 3, 4, &c. on the supposed radius next below B, do represent .1, .2, .3, .4, &c. tenths of an unit; and on the second

second radius below B, they represent .01, .02, .03, .04, &c. or 1. 2. 3. 4. &c. hundredths of an unit; and so on proportionably, as the distance of the radius on which any number is supposed to be found, is from the radius first appropriated.

N. B. The same is to be understood with respect to the radius A, and the radii both above and below it.

Example. Let the prime 5 on A represent 50; then will the prime 6, 7, 8, 9, &c. represent 60, 70, 80, 90, &c. The like primes on the supposed radius next above A, will now represent 600, 700, 800, 900, &c. and on the next radius above that they will represent 7000, 8000, 9000, &c.

At the same time the like primes on the supposed radius below A, will represent 6, 7, 8, 9, &c. units, and those on the second radius below will represent .6, .7, .8, .9, &c. tenths and so on.

Again, let the intermediate 245 on C represent 24.5: then will the same intermediate on the supposed radius above C, represent 245.; and on the second radius above C, the same intermediate will represent 2450.

At the same time the said intermediate 245 on B, doth represent 2.45; on the supposed radius next below B .245, and on the supposed radius next below that, the said intermediate will represent .0245, and so on.

The

The ſame is to be underſtood of any other prime or intermediate.

SECT. III. *Of the Tranſlations of Primes and Intermediates.*

Under this article I mean to ſhew, how any number ſuppoſed to be repreſented on any imaginary radius, may be transferred to one or other of the radii A, B, or C.

1. Place the radius B direct, and let it repreſent numbers conſiſting of two integral places, and and let the radius A repreſent numbers of one integral place.

Now from what hath been ſaid in the foregoing chapters, it is evident, that each radius above and below A doth repreſent numbers conſiſting of one place leſs than its reſpective collateral.

2. Suppoſe the radius A to repreſent numbers conſiſting of two integral places, and let the radius B repreſent fractions of the firſt order, viz. tenths, that is 2. places leſs.

It is evident, in this caſe, that the primes and intermediates on each radius above and below A, do repreſent numbers conſiſting of two places more than the primes and intermediates, on their reſpective collaterals.

Again, place the prime 2 on A to the intermediate 25 on B, and ſuppoſe the former to repreſent the number 2. viz. 2 units, and the latter 25. units.

Then

Then the radius A being determined to represent numbers consisting of one integral place only, and the radius B of two, it is evident the prime 4 or A, doth represent the number 4. and doth stand against the prime 5 on B, which now naturally represents 50. Now, if I suppose the prime 4 or A to become 40, viz. one place more than the said prime doth really represent, it must be supposed to be found on the radius next above A, and consequently the prime 5, against which it stands, must also be supposed to be on C its collateral, which doth represent one place more than the radius B, and which therefore doth now represent 500, viz. one place more than 40, as 25. is one place more than 2.

COROLLARY.

Hence it follows, that the number of places in any number, represented by any prime or intermediate on any imaginary radius above or below A, bears the same proportion to the number of places in any number represented by any prime or intermediate on its collateral, as the number of places in any number represented on A, doth to the number of places represented by its collateral B.

CHAP. IV.

Of the Nature and Use of the Collateral Radius.

SECT. I. *Of the Nature of the Collateral Radius.*

I HAVE already given a general deſcription of the collateral radius, but as it is abſolutely neceſſary in every calculation by the inſtruments, to have a right and juſt underſtanding thereof, I ſhall here ſhew what is more particularly meant thereby.

Firſt then, it is to be obſerved, that in every operation by the inſtruments, three numbers are always ſuppoſed to be given, and a fourth required.

Now the firſt of any three given numbers may be called the prime number; and that radius whereon it is taken, may be called the prime radius.

2. One of the other two numbers, no matter which, muſt always be placed right againſt the prime number, and when thus placed, is called the ſecond or collateral number; and that radius whereon this ſecond number is taken, is called the collateral radius, or the *collateral.*

SECT. II. *Uſe of the Collateral Radius.*

N. B. On the right underſtanding of this radius in a great meaſure depends the whole myſtery

of calculation by the inſtrument; for by the line A, and alſo I, the fourth number hath 3 varieties, by the line D it hath 5, and by the line E 7.

Thus by the line A, it may either fall on the collateral, or the next radius above the collateral, or on the next below it.

1. Place 2 on B, to 1 on A, and let B be collateral. Now, if the third number be repreſented by any prime or intermediate between the prime 1 incluſive, and the prime 5 excluſive on A, then the fourth number will be found on B the *collateral*, and ſo the anſwer is ſaid to fall *on the collateral.*

But if the third number be repreſented by the prime 5, or any other prime or intermediate above 5 on A, then the fourth number will be found on C, and the anſwer will therefore be ſaid to fall *above the collateral.*

2. Place 4 on C to 10 on A, and let C be the collateral; now, if the third number be repreſented by the intermediate 25, or any prime or intermediate above it on A, then will the fourth number be found on C the *collateral*, and the anſwer be ſaid to fall *thereon.*

But if the third number be repreſented by the prime 1 or 2, or by any intermediate below the intermediate 25 on A, then will the fourth number be found on B, and the anſwer ſaid to fall off *below* the *collateral.*

Hence

Hence observe,

1. If $\left\{\begin{matrix} B \\ C \end{matrix}\right\}$ be collateral, the answer will fall thereon, or off $\left\{\begin{matrix} \text{above} \\ \text{below} \end{matrix}\right\}$ it.

2. If the first or prime number be taken on B or C, then A must be the collateral.

And if B be prime, then, because the first and third numbers must always be taken on one and the same radius, the answer will fall on the collateral, or off *below* it.

Thus, place 4 on B to 1 on A, and let B be prime radius, then if the third number be represented by any prime or intermediate above the prime 4 on B, the answer is said to fall *on* the collateral A.

But if the third number be represented by any prime or intermediate below the prime 4, the answer doth fall off *below* the collateral.

From what hath been said, it is evident, that if C be prime radius, the answer must fall on the collateral, or off *above it*. See chap. III. sect. 1.

N. B. The varieties of D and E will be shewn in their proper places.

CHAP. V.

Of the Dispoſition of Primes and Intermediates on the Lines A, B, and C, with the Manner of working of ſimple Proportions thereby, &c.

SECT. I. *Of the Dispoſition of Primes and Intermediates on the Lines A, B, and C.*

THE primes and intermediates on theſe lines are diſpoſed in ſuch a manner, as that, if you place the firſt of any three given numbers on any radius, right againſt the ſecond, on any other radius; then againſt the third number on the prime or firſt radius, you will have a fourth geometrical proportional to the ſaid three given numbers: that is a number which ſhall bear the ſame proportion to the third number, as the ſecond doth to the firſt. Hence,

SECT. II. *Of the Manner of working Proportions by the Lines A, B, C.*

RULE.

Set the firſt of the three given numbers, on any radius, againſt the ſecond on ſome other radius; then againſt the third number on the prime radius, you will have the fourth proportional to the ſaid given numbers.

Thus,

Thus, ſet 2 on A to 3 on B; then againſt 4 on A, is 6 on B, the fourth proportional to 2. 3. and 4.

Or, if you ſet 2 on C to 3 on A; then againſt 4 on C is 6 on A, the fourth proportional.

Lemma I.

In every four geometrical proportional direct, the product or rectangle made of the two mean or middle numbers, will be equal to the product or rectangle made of the two extremes. Thus in the above example.

Set 2 on A to 4 on B; then againſt 3 on A is 6 on B, as in the firſt example. And

Set 2 on C to 4 on A; then againſt 3 on C is 6 on A, as in the ſecond example.

Thus characterd.

$$\overset{2}{A} : \overset{4}{B} :: \overset{3}{A} : \overset{6}{B}$$

and

$$\overset{2}{C} : \overset{4}{A} :: \overset{3}{C} : \overset{6}{A}$$

or,

$$\overset{2}{A} : \overset{3}{B} :: \overset{4}{A} : \overset{6}{B}$$

and

$$\overset{2}{C} : \overset{3}{A} :: \overset{4}{C} : \overset{6}{A}$$

Hence obſerve, either of the two means may be made the ſecond term in any proportion.

SECT. III. *Of finding the Number of Places in in the Fourth Proportional.*

Because the first and third numbers in every proportion is supposed to be taken on the same radius, it is evident, that if the first and third numbers in any proportion consist of equal places, and the answer falls on the collateral, the fourth proportional will consist of equal places with the second number. (See chap. III. sect. 2.) Hence,

If the first and third numbers in any proportion are unequal, then,

As many places as the third hath more or less than the first, so many places will the fourth have more or less than the second, if it falls on the collateral, (by corol. chap. III. sect. 3.)

Or, as many places as the second number hath more or less than the first, so many places will the fourth have more or less than the third, if it falls on the collateral. (See lemma, chap. V.)

N. B. If the answer falls above the collateral, it will have one place more; if below, one place less, in all cases, than if it falls on it. (See *N. B.* chap. III. sect. 2.)

Lemma 2.

When three numbers are in geometrical proportion, the square of the mean or middle number, will be equal to the product or rectangle made of the two extremes. Hence,

To find the third geometrical proportional direct to any two given numbers,

Rule.

Rule.

Set the first number on any radius to the second on any other radius; then against the second on the prime radius, is the third proportional.

Thus, set 2 on A to 4 on B; then against 4 on A, is 8 on B and C.

CHAP. VI.

Of the Manner of performing Multiplication, Division, and the Rule of Three Direct, by the Lines A, B, C.

OBSERVE. From the nature of the lines, the product of every multiplication, the quotient of every division, and the answer in the rule of three, must be each a fourth geometrical proportional direct to some three given numbers.

Hence, I. For Multiplication.

If unity, or 1, be made the first term in the proportion, and the given factors the two means, the fourth proportional will be the product of the said two factors. (See lemma 1. chap. V.)

Thus, let the given factors be 8 and 6.

	1		8.		6.		48. the 4th prop.
Then it will be	A	:	B	::	A	:	C.

Extremes. Means.

Now, $1.\times 48.=8.\times 6.=48$ the product.

II. For Division.

If the given divisor be made the first term in the proportion, and unity and the divisor the two means, the fourth proportional will be the quotient of such division. (See lemma as above.)

Thus, let 6 be the divisor and 48 the dividend.

	6.	1.	48.	8. the 4th prop.
Then it will be	B :	A ::	C ;	A natural

Extremes. Means.

$6\times 8=1.\times 48=48.$ and $48\div 6=8.$

III. For the Rule of Three Direct.

If the divisor be made the first term in the proportion, and the other two numbers the two means, the fourth proportional will be the answer. (See as above.)

Thus, let the divisor be 4 and the other numbers 6 and 8.

	4	6	8	12 the 4th propor.
Then	A :	B ::	A :	C.

Extremes. Means.

$4\times 12=6\times 8=48.$ and $6\times 8\div 4=12$ Ans.

SECT. II. *Of finding the Number of Places in the Product of any two Factors.*

From what hath been said (chap. V. sect. 3.) it appears, that if the third number in any proportion consisteth of one integral place, and the answer falls on the collateral, it will consist of equal places with the second number.

But

But it is evident in this case, that the second number consisteth of one place less than the sum of the number of places in the second and third numbers in the proportion, viz. the two factors.

Hence, if the answer falls on the collateral, it will consist of one place less than the sum of the number of places in the two factors. (See corol. chap. III. sect. 3.) If it falls above, it will have equal places therewith. (See *N. B.* chap. III. sect. 2.)

Example 1. Given the factors 3.4 and 25. what is their product.

1.	3.4	25.	85.	answer.
A :	B ::	A :	B	

Answer falls on the collateral; therefore it hath one place less than the sum of the number of places in both factors.

Or thus natural.

1.	25.	3.4	85.
A :	B ::	A :	B

Examp. 2. What is the product of 4.8 by 25.?

1	4.8	25.	120.	answer.
A :	B ::	A :	C	

The answer falls above the collateral; thence it hath as many places as are in both factors.

Or thus natural.

1	25.	4.8	120.
A :	B ::	A :	C

Hence observe, if either of the given factors consisteth of one place of integers, make that the

the third number in the proportion, and the anſwer will be natural.

Obſerve alſo, if either of the factors be a fraction of the firſt order, then

Let 10. on A repreſent unity; then will the product be found natural.

Examp. 1. What is the product of 36 by .75?

1.0	36	.75	27. anſwer.
A :	C ::	A :	C

Examp. 2. What is the product of 36 by .25?

1.0	36	.25	9. anſwer.
A :	C ::	A :	B

Note. If the fraction be of any other order, then

As many cyphers as are prefixed thereto; ſo many places will the anſwer have leſs than natural.

Thus, if the fraction in the firſt example had been .075, and in the ſecond .0025, the anſwers would have been 2.7 and .09.

SECT. III. *Of finding the Number of Places in the Quotient ariſing from the dividing of one Number by another.*

From what hath been ſaid in the former ſection, it appears, that

If unity be made the ſecond number in the proportion, and the diviſor and dividend conſiſts of equal places, if the anſwer falls on the collateral it will conſiſt of one integral place, viz. one

place

place more than the difference of places in the divisor and dividend: also,

If the dividend be made the second number in the proportion, and doth consist of equal places with the divisor, if the answer falls on the collateral, it will consist of equal places with the third number, viz. one place more than the said difference of places.

Hence, if the answer falls on the collateral, it will consist of one place more than the difference between the number of places in the divisor and those in the dividend. (See chap. V. sect. 3.)

If the answer falls below, it will have equal places with the said difference. (See ditto, also *N. B.* chap. III. sect. 2.)

Examp. 1. What is the quotient of 85. divided by 2.5?

2.5		1		85.		34.
A	:	C	::	A	:	C

The differences of places in the divisor and dividend = 1. answer falls on therefore it hath 2 places.

Examp. 2. Divide 120. by 2.5.

2.5		1		120.		48.
A	:	C	::	A	:	B

The difference of places = 2 answer falls below.

Observe, if the divisor consisteth of one integral place, make 1. the third number in the proportion, then will the answer be natural.

Observe also, if the divisor be a fraction, let 10 on A represent 1. and make it the third term in

in the proportion, then if the fraction be of the first order, the anſwer will be natural.

Examp. 1. Divide 27. by .75.

.75 27. 1.0 36. anſwer.
A : C :: A : C, on,

Examp. 2. Divide 9. by .25.

.25 9 1.0 36. anſwer.
A : B :: A : C above.

Compare the examples in this ſection, with thoſe of multiplication.

N. B. If the diviſor be a fraction of any other order, then

As may cyphers as are prefixed therein, ſo many places will the anſwer have more than natural.

N. A. Multiplication may be performed by the prime C, and diviſion by the prime A; regard being had to the reſpective collaterals. (See chap. IV. ſect. 2.)

CHAP. VII.

Tables of Factors, Diviſors, and Gauge Points, with their Characteriſticks and Uſes.

1. *Officer's Tables.*

TABLE I. DIVISORS on upper A of the *officer*'s inſtrument for gauging of right lined areas and ſolids, at *one operation.*

No.

No.	Divisors.	Charact.	Use.	
1	2150.	MB::	For Malt - - -	Bushels.
2	227.	MT.·.	Mash Tun - -	Gallons.
3	2300.*	SF::*	Starch Fat - -	Bushels.
4	231.	WG.·.	Wine - - -	Gallons.
5	25.56	WSS:	White Soft Soap	Pounds.
6	25.67	GSS:*	Green ditto - -	Ditto.
7	268.	MG.·.	Malt - - -	Gallons.
8	27.14	HS:	Hard Soap - -	Pounds.
9	282.	AG.·.	Ale - - -	Gallons.
10	30.28	Tp:	Tallow gross -	Pounds.
11	31.40	Tpn:	Tallow neat - -	Ditto.
12	34.81	GS:	Green Starch -	Ditto.
13	40.5	DS:	Dry Starch - -	Ditto.

* The divisor SF. (No. 3.) for want of room is put on radius I 2, and marked SF□::; also divisor GSS (No. 6.) on radius I, and marked GSS:□

N. B. Divisor MG.·. for want of room is omitted.

TABLE II. Divisors on the inverted radius I of the *officer's* instrument for gauging of circular and elliptical areas and solids at *one operation*.

No.	Divisors.	Charact.	Use.	
14	11.68	PG:	For Plate Glass - -	Pounds.
15	13.39	CG:	Crown ditto - -	Ditto.
16	25.67	GSS:	Green Soft Soap□	Ditto.
17	2928.	SF::	Starch Fat -	Bushels.
18	32.54	WSS:	White Soft Soap	Pounds.
19	34.55	HS:	Hard Soap -	Ditto.
20	38.55	Tp:	Tallow neat -	Ditto.
21	39.97	Tpn:	Tallow gross -	Ditto.

TABLE

3

TABLE III. Divisors on the inverted radius I 2 of the officer's instrument for gauging of circular and elliptical areas, and solids at *one operation.*

No.	*Divisors.*	*Charact.*	*Use.*	
22	10.77	FG:	For Flint Glass - -	Pounds.
23	12.96	GB:	Bottle ditto -	Ditto.
24	2300.	SF□:	Starch Fat - -	Bushels.
25	2738.	MB::	Malt - - -	Ditto.
26	289.	MF.·.	Mash Tun - -	Gallons.
27	294.1	Wg.·.	Wine - -	Ditto.
28	32.68	GSS:	Green Soft Soap	Pounds.
29	359.	Ag.·.	Ale - - -	Gallons.
30	44.32	GS:	Green Starch -	Pounds.
31	51.3	DS:	Dry ditto -	Ditto.
		Factor on radius A		
32	3.141	OC.	Circumf. of a circle, diamet.	1.

TABLE IV. Divisors which may be put on the inverted line I of the officer's instrument for gauging of ale, polygons and their prisms.

No.	*Divisors.*	*Charact.*	*Use.*
33	108.5	6gn.·.	For the Hexagon, Side given.
34	14.02	16gn:	Ekdecagon
35	163.9	5gn.·.	Pentagon
36	25.18	12gn:	Dodecagon
37	30.11	11gn:	Endecagon
38	3543.	⊙c::	Circle, Circ. } given
39	359.	⊙d.·.	Ditto, Diam. } given
40	36.66	10gn:	Decagon, Side
41	45.61	9gn:	Nonagon
42	48.40	8gn:	Octagon
43	651.2	3gn.·.	Trigon
44	77.6	7gn:	Heptagon
45	8.932	20gn.	Icosagon

TABLE

TABLE V. Divisors which may be put on the radius I 2 of the *officer's* instrument for wine polygons and their prisms.

No.	*Divisor.*	*Charact.*	*Use.*
46	11.48	16gn:	For the Ekdecagon, Side given
47	134.2	5gn.·.	Pentagon
48	20.63	12gn:	Dodecagon
49	24.66	11gn:	Endecagon
50	294.	Od.·.	Circle, Diam. } given
51	2902.	Oc::	Ditto, Circ. } given
52	30.02	10gn:	Decagon, Side
53	37.36	9gn:	Nonagon
54	47.84	8gn:	Octagon
55	533.4	3gn.·.	Trigon
56	63.56	7gn:	Heptagon
57	7.317	20gn.	Icosagon
58	88.91	6gn:	Hexagon

N. B. The dots or points immmediately preceding or following the characteristicks of the factors, divisors and gauge points, denote their value; thus,

1. If a point or points stand at the right-hand of any factor, &c. it denotes such factor, &c. to be an integral or mixed number; and the number of points shew the number of integral places it consisteth of.

2. If a point or points stand at the left-hand of any factor, &c. it denotes it to be a fraction; and the number of points shews of what order the said fraction is.

N. A. The factors, divisors and gauge poinst on the instruments, are charactered in the same manner as in these tables.

TABLE

TABLE VI. Divisors which may be put on upper A of the *officer's* instrument, for malt polygons and their prisms.

No.	*Divisors.*	*Charact.*	*Use.*
59	106.9	16gn.	For the Ekdecagon, Side given
60	1249.	5gn:	Pentagon
61	192.	12gn.	Dodecagon
62	229.6	11gn.	Endecagon
63	27023.	⊙c:	Circle, Circ. } given
64	2738.	⊙d:	Ditto, Diam. }
65	279.4	10gn.	Decagon, Side
66	347.8	9gn.	Nonagon
67	445.3	8gn.	Octagon
68	4966.	3gn:	Trigon
69	591.7	7gn.	Heptagon
70	68.11	20gn:	Icosagon
17	827.7	6gn.	Hexagon.

TABLE VII. Gauge points on the line D of the *officer's* instrument for gauging of circular areas and cylinders.

No.	*G. P.*	*Charact.*	*Use.*	
72	17.15	WG:	For Wine - - - - -	Gallons.
73	17.3	MT:	Mash Tun - - -	Ditto.
74	18.49	MG:	Malt - - - -	Ditto.
75	18.95	AG:	Ale - - - - -	Ditto.
76	3.282	FG.	Flint Glass - -	Pounds.
77	3.418	PG.	Plate ditto - - -	Ditto.
78	3.6	GB.	Bottle ditto - -	Ditto.
79	3.66	CG.	Crown ditto - -	Ditto.
80	52.32	MB:	Malt - - -	Bushels.
81	5.704	WSS.	White Soft Soap -	Pounds.
82	5.717	GSS.	Green ditto - -	Ditto.
83	5.878	HS.	Hard Soap - - -	Ditto.
84	6.21	Tp.	Tallow gross - -	Ditto.
85	6.323	Tpn.	Tallow neat - -	Ditto.
86	6.657	GS.	Green Starch - -	Ditto.
87	7.162	DS.	Dry ditto - - -	Ditto.

2. Arti-

2. *Artificer's Tables.*

TABLE VIII. Factors on the lower edge of the ſlide B, of the *artificer's* inſtrument, to be uſed with A in the proportions of the ſides of ſuperficies inſcribed in a circle, &c.

No.	*Factors.*	*Char.*		*Uſe.*			
88	.2250	.Sic	For the ſide of	Square	inſcrib'd	cir-cle	circum ference given.
89	.2756	.△c		Triangle	in a		
90	.2821	.Sec		Square equal to a			
91	.7071	.Sid		Square	inſcrib'd	cir-cle	diame-ter given.
92	.8660	.△d		Triangle	in a		
93	.8862	.Sed		Square equal to a			

TABLE IX. Factors and diviſors on lower A of the *artificer's* inſtrument, to be uſed with B and C in meaſuring of ſuperficies.

N°.	*Fact. & Div.*	*Char.*	*Uſe.*
94	10.	L□:	For Rectangular land, ſtatute acre
95	12.	12:	Board, &c.
96	20.	L△:	Triangular land, ſtatute acre
97	3.141	Oc.	Circumf. of a circ. diam. given and è cont.
98	.4726	.L24	Reduc. cuſtom. land acre, of 24 pch. to ſtatute acre.
99	.6173	.L21	Ditto - - - - - - 21
100	.7854	.td	Reducing timber meaſure to cuſtomary
101	.8462	.L18	Reduc. cuſtom. land of 18 pch. to ſtatute
102	9.	□Y^{d}.	Square Yards
103	100.	□.·.	Great Square of 100 feet.

TABLE X. Divisors on upper A of the *artificer's* instrument, to be used with A, B and C, in measuring of superficies and solids at one operation.

N°.	*Divis.*	*Char.*	*Use.*
104	100.	Shw.·.	For Burthen of ships of war
105*	12.	Bst:	Stock of boards
106*	12.	GL:	Glass lights, feet, dimen. inch. & feet
107	1.273	⊙d.	Ellipsis, circ. or prism, diamet. given
108	128.	CW.·.	Cord wood
109†	144.	□Tim.·.	Rectangular timber
110†	144.	GL.·.	Glass lights, dimensions, inches
111†	1440.	K::	Hundreds of sawing
112	1728.	RM::	Roods of marle
113	18.	ST:	Tons of soil
114	1810.	⊙Tim::	Circular or ellipt. timber, circ. given
115	183.3	⊖Tim.·.	Ditto - - - - - - diam.
116	408.3	BW.·.	Brickwork in perch. dimen. ft. & pts
117	9.	Polpr.	Superficies of polygonal prisms
118	93.64	Brk°:	Numb of bricks in walling, dim. ft.
119	94.	Shst:	Burthen of ships, statute } dimensions feet.
120	95.	Shm:	Ditto of merchant-men } dimensions feet.

N. B. Any of the above divisors may be put on either of the inverted lines, viz. I, or I 2, if want of room or any conveniency require it.

N. A. If occasion requires, factors, divisors and gauge points, may be translated into each other, thus;

1. For Factors and divisors.

Divide unity by the one, and the quotient will be the other.

* Numbers 105 and 106, are both found at the same point.

† Numbers 109, 110 and 111, are all found at the same point.

2. For

2. For Divisors and Gauge Points.

The square root of any divisor is its equivalent gauge point.

The square of any gauge point is its equivalent divisor.

TABLE XI. Divisors *which may be put* on radius I, of the *artificer*'s instrument, for finding the weight of right lined prisms, of divers bodies, in the great hundred, dimensions being taken in feet and decimal parts.

N°.	*Divisors.*	*Charact.*	*Use.*
121	1.739	B.	For Box
122	1.932	O.	Oak
123	3.229	F.	Fir
124	.661	.M	Marble
125	.7168	.S	Stone
126	.9562	.A	Alabaster.

TABLE XII. Divisors *which may be put* on radius I 2 of the *artificer*'s instrument, for finding the weight of circular or elliptical prisms, of divers bodies, in the great hundred, dimensions being taken in feet and decimal parts.

N°.	*Divisors.*	*Charact.*	*Use.*
127	1.217	A.	For Alabaster
128	2.265	B.	Box
129	2.460	O.	Oak
130	4.112	F.	Fir
131	.8416	.M	Marble
132	.9126	.S	Stone

TABLE XIII. Gauge points on the line D of the *artificer*'s instrument, to be used with B and C, in measuring of polygons and their prisms.

Nº.	G. Points.	Charact.		Use.
133	13.54	⊖d:	For the	Circle, diameter given
134	18.23	3gn:		Trigon, side
135	2.135	20gn.		Icosagon
136	2.667	16gn.		Ekdecagon
137	3.586	12gn.		Dodecagon
138	3.921	11gn.		Endecagon
139	42.53	⊙c:		Circle, circumference
140	4.326	10gn.		Decagon, side
141	4.826	9gn.		Nonagon
142	5.461	8gn.		Octagon
143	6.295	7gn.		Heptagon
144	7.445	6gn.		Hexagon
145	9.148	5gn.		Pentagon

TABLE XIV. Factors on the upper edge of the slide B of the *artificer*'s instrument, to be used with D, in finding the superficial contents of polygons in square yards, a side being taken in feet and decimal parts.

Nº.	Factors.	Charact.		Use.
146	1.04	11gn.	For the	Endecagon, side given
147	1.244	12gn.		Dodecagon
148	.1911	.5gn		Pentagon
149	2.234	16gn.		Ekdecagon
150	.2886	.6gn		Hexagon
151	3.507	20gn.		Icosagon
152	.4037	.7gn		Heptagon
153	.04811	:3gn		Trigon
154	.5363	.8gn		Octogon
155	.6868	.9gn		Nonagon
156	.8549	.10gn		Decagon
157	.08726	:⊖d		Circle, diameter given.

TABLE

TABLE XV. Factors on the lower edge of slide C, or C 2, of the *artificer's* instrument, to be used with D, in finding the superficial content of the platonicks, or five regular bodies in square yards, a side being taken in feet and decimal parts.

N°.	*Factors.*	*Charact.*	*Use.*
158	.1924	.4rn	For the Tetraedron
159	2.2938	12rn.	Dodecaedron
160	.349	.Spd	Sphere, diameter given
161	.03563	.Spc	Ditto, circumference
162	.3849	.8rn	Octaedron
163	.6666	.6rn	Hexaedron
164	.9622	.20rn	Icosaedron

TABLE XVI. Factors on upper edge of radius E of the *artificer's* instrument, to be used with D, in finding the solidities of the five platonicks.

N°.	*Factors.*	*Charact.*	*Use.*
165	1.	6rn.	For the Hexaedron
166	.1178	.4rn	Tetraedron
167	.01688	:Spc	Sphere, circum. given
168	2.221	20rn.	Icosaedron
169	.4714	.8rn	Octaedron
170	.5236	.Spd	Sphere, diameter
171	7.657	12rn	Dodecaedron

TABLE XVII. Factors on upper E 2 of the *artificer's* instrument, to be used with D, for finding the weight of the platonicks in common stone, in pounds Averdupoise, a side being taken in inches and decimal parts.

No.	*Factors.*	*Charact.*		*Use.*
172	.01065	:3rn	For the	Tetraedron
173	.001526	.·.Spc		Sphere, circumference
174	.02008	:20rn		Icosaedron
175	.04262	:8rn		Octaedron
176	.04734	:Spd		Sphere, diameter
177	.6924	.12rn		Dodecaedron
178	.09041	:6rn		Hexaedron

TABLE XVIII. Factors on lower E 3 of the *artificer's* instrument, for finding the weight of the platonicks of box, &c.

No.	*Factors.*	*Charact.*		*Use.*
179	.01756	:8rn	For the	Octaedron, side given
180	.01950	:Spd		Sphere, diameter
181	.02725	:6rn		Hexaedron
182	.2852	.12rn		Dodecaedron
183	.004390	.:.4rn		Tetraedron
184	.0006290	::Spc		Sphere, circumference
185	.08274	:20rn		Icosaedron

TABLE XIX. Factors on E 4 of the *artificer's* instrument, for the weight of platonicks of marble, &c.

No.	*Factors.*	*Charact.*		*Use.*
186	.01155	:4rn	For the	Tetraedron, side given
187	.001655	.·.Spc		Sphere, circumference
188	.2177	:20rn		Icosaedron
189	.04622	:8rn		Octaedron
190	.05134	:Spd		Sphere, diameter
191	.7508	.12rn		Dodecaedron
192	.09805	:6rn		Hexaedron

Factors

Factors on E 5.

For the weight of the ſphere in iron and lead.

No.	*Factors.*	*Charact.*	*Uſe.*	
193	.1447	.Spd.I	For Iron	diameter taken
194	.2144	.Spd.L	Lead	

CHAP. VIII.

Of Multiplication, Diviſion, and the Rule of Three Direct, by the Line A.

SECT. I. *Of Multiplication.*

1. *By the Prime Radius A.*

PROPORTION.

AS unity (or 1.) on A,
Is to either of the factors on B;
So is the other factor on A,
To the product on B or C.

See Multiplication, *chap.* VI.

That is, place one of the factors on B, againſt unity (or 1) on A; then againſt the other on A, is the product on B or C.

To find the number of places in the product.

RULE.

If the anſwer falls above the collateral, (viz. on C.) it will conſiſt of as many places as the ſum of the number of places in both the factors.

If the anſwer falls on the collateral, (viz. on B) it will have one place leſs. *See chap.* VI. *ſect.* 2.

To find the ſum of the number of places in any two given factors.

I. If the given numbers are both integral, or both mixed; or one integral and the other mixed, or one of them a fraction of the firſt order,

The ſum of the number of places in both is equal to the ſum of the number of integral places in both given numbers.

II. If one of them be an integer or mixed number, and the other a fraction of any other order, then,

1. If the number of integral places in the one, exceed the number of cyphers prefixed in the other; deduct as many places from the ſaid integral or mixed number as there are cyphers prefixed, and the remainder will be the ſum of the number of places in both.

2. If the number of cyphers prefixed be equal to the number of integral places, the ſum of the number of places in both will be negative, and will be expreſſed by a fraction of the firſt order.

Hence, if the anſwer falls on B, it will be a fraction of the ſecond order. *See the foregoing Rule.*

3. If the cyphers prefixed in the one exceed the number of integral places in the other, the ſum of the number of places will be expreſſed by a fraction, having as many cyphers prefixed as the exceſs is.

Hence, if the anſwer falls on B, it will have one cypher more prefixed than the ſaid exceſs.

III. If

III. If both numbers are fractions, the sum of the number of places in both will be expressed by a fraction, with as many cyphers prefixed as are prefixed in both.

Hence, if the answer falls on B, it will have one cypher more prefixed, than the sum of the number of cyphers prefixed in both.

Examp. 1. Multiply 25. by 14.

1. 25. 14. 350. Answer
A : B :: A : B on the collateral

Examp. 2. Multiply 2.4 by 75.

1. 2.4 75. 180. answer
A . B :: A : C above

Examp. 3. Multiply 2.4 by 25.

1. 2.4 25: 60.
A : B :: A : B

Note. If either of the factors consisteth but of one integral place, make that the third number in the proportion, and the answer will be natural. Thus in the last example.

1 25. 2.4 60.
A : B :: A : B

Examp. 4. Multiply 48. by .25.

1 48. .25 12.
A : B :: A : C above

Examp. 5. Multiply 32.5 by .04.

1 .04 32.5 1.3
A : B :: A : C above

Examp.

Examp. 6. Multiply 3.4 by .0025.

1 3.4 .0025 .0085
A : B :: A : B

Examp. 7. Multiply .04 by .075.

1. .04 .075 .003
A : B :: A : C above

Note. When either of the factors is a fraction of the first order, let 10 on A represent unity, and make the said fraction the third number in the proportion, then will the answer be found natural: thus in the fourth example.

1.0 48. .25 12.
A : C :: A : C natural

N. A. If the fraction be of any other order, suppose it to be of the first order, and find the answer thereto natural; then as many cyphers as are prefixed in the fraction, so many places will the answer have less than the abovesaid answer.

2. *By the Prime Radius C.*

N. B. In all oblique positions of the slides B and C, that part of B which stands below the radius A, is represented by its like part of C; and that part of radius C, which stands above A, is represented by its like part of B. *See chap.* III. *sect.* 1.

Hence, because the third number in every proportion is supposed to be found on the prime radius, it follows, that if it be found on B, the answer falls off above the collateral. *See* Observations, *chap.* IV. *sect.* 2.

Examp.

Examp. 1. Multiply 35. by 24.

1.		35		24		840 answer
C	:	A	::	C	:	A

Examp. 2. Multiply 44. by 5.

1		44		5		220. answer
C	:	A	::	B	:	A above

Reduction of Fractions by Multiplication.

RULE.

Multiply the given fraction by a number equal to the number of parts, into which the given integer is, by the question, supposed to be divided, and the product will be the equivalent.

By the Prime A.

Examp. 1. What is the value, in shillings, of .75 parts of a pound sterling?

1.0		20		.75		15. answer
A	:	C	::	A	:	C natural

Examp. 2. How many inches is .75 parts of a foot?

1.0		12.		.75		9. answer
A	:	C	::	A	:	B natural

SECT. II. *Of Division.*

1. *By the Prime Radius A.*

PROPORTION.

As the divisor on A,
Is to unity (or 1.) on C;
So is the dividend on A,
To the quotient on B or C.

See Division, *chap.* VI.

To

To find the number of places in the quotient.

RULE.

If the anſwer falls below the collateral, (viz. on B) the number of places in the quotient, will be equal to the difference of the number of places in the diviſor, and thoſe in the dividend.

If the anſwer falls on the collateral, (viz. C.) it will have one place more than the ſaid difference. *See chap.* IV. *ſect.* 3.

To find the difference of places in any two given numbers.

I. If both the given numbers are integral or mixed; or one an integral, and the other a mixed number,

The difference of the number of places in the ſaid numbers will be equal to the difference of the number of integral places in both.

II. If one of the given numbers be integral or mixed, and the other a fraction, then

1. If the fraction be of the firſt order, the difference of the number of places in the ſaid numbers, will be equal to the number of integral places in the whole or mixed number.

2. If the fraction be of any other order, the difference of places will be equal to the ſum of the number of integral places in the one added to the number of cyphers prefixed in the other.

III. If both numbers are fractions, the difference of places, will be equal to the difference of the number of cyphers prefixed in the ſaid numbers.

Examp. 1. Divide 150. by 25.

25. 1. 150. 6. answer
A : C :: A : B below

Examp. 2. Divide 54. by 2.25.

2.25 1. 54 24.
A : C :: A : C

Examp. 3. Divide 28.5 by .75.

.75 1. 28.5 38.
A : C :: A : B below

Examp. 4. Divide 6.5 by .025.

.025 1. 6.5 260.
A : C :: A : C

Examp. 5. Divide .075 by .0025.

.0025 1. .075 30.
A : C :: A : C

Note. If the divisor consisteth of but one integral place; or is a fraction of the first order, the quotient may be found natural by the following

PROPORTION.

As the divisor on A,
Is to the dividend on B or C;
So is unity on A.
To the quotient on B or C.

See chap. VI. *sect.* 3.

Examp. 1. Divide 180. by 4.5.

4.5 180. 1. 40.
A : C :: A : B

Examp. 2. Divide 36 by .15.

.15 36 1.0 240.
A : B :: A : C

Note.

Note. If a leſſer number be divided by a greater, the quotient will be a fraction, the value whereof will be found by the following

RULE.

If the anſwer falls below the collateral, the number of cyphers to be prefixed, will be equal to the difference of places in the diviſor, and thoſe in the dividend.

If it falls on the collateral, it will have one cypher leſs.

Examp. 1. Divide 2. by 40.

40. 1. 2. .05 anſwer
A : C :: A : B below

Examp. 2. Divide 4. by 20.

20 1. 4 .2 anſwer
A : C :: A : C

N. B. Diviſion may be performed by the prime radius B or C, regard being had to the anſwers falling on or off the collateral. *See the N. B. on* Multiplication *by the prime C.*

How to reduce a vulgar fraction to its equivalent decimal.

RULE.

Divide the numerator of the given fraction by its denominator.

PROPORTION.

As the denominator on C,
Is to unity on A;
So is the numerator on B or C,
To the decimal on A.

Examp.

Examp. 1. Reduce $\frac{1}{4}$.

4.		1.0		1		.25 answer	
C	:	A	::	C	:	A	

Examp. 2. Reduce $\frac{3}{4}$.

4		1.0		3.		.75
C	:	A	::	C	:	A

Examp. 3. Reduce $\frac{3}{5}$.

5.		1.0		3.		.6
C	:	A	::	C	:	A

Examp. 4. Reduce $\frac{6}{40}$.

40.		1.0		6		.15
C	:	A	::	B	:	A

Examp. 5. Reduce $\frac{3}{40}$.

40		1.0		3		.075
C	:	A	::	C	:	A below

SECT. III. *Of the Rule of Three Direct.*

PROPORTION.

As the first number
Is to the second;
So is the third
To the fourth.

See chap. V. *sect.* 1, 2.

To find the number of places in the answer.

RULE.

As many places as the second number in the proportion hath $\left\{\begin{matrix}\text{more}\\\text{less}\end{matrix}\right\}$ than the first; so many will

will the anſwer have $\left\{\begin{matrix}\text{more}\\ \text{leſs}\end{matrix}\right\}$ than the third, if it falls on the collateral.

If the anſwer falls $\left\{\begin{matrix}\text{above}\\ \text{below}\end{matrix}\right\}$ the collateral $\left\{\begin{matrix}\text{add}\\ \text{deduct}\end{matrix}\right\}$ one place.

Or thus:

As many places as the third number hath $\left\{\begin{matrix}\text{more}\\ \text{leſs}\end{matrix}\right\}$ than the firſt; ſo many will the anſwer have $\left\{\begin{matrix}\text{more}\\ \text{leſs}\end{matrix}\right\}$ than the ſecond; if it falls on the collateral. *See chap.* V. *ſect.* 3.

1. *By the Prime Radius A.*

Examp. 1. Given the numbers 35. 5. and 315.

35. 5. 315. 45 anſwer
A : B :: A : B

Examp. 2. Given 25. 75. and 8.

25. 75 8. 24.
A : B :: A : C above

Examp. 3. Given 75. 2.5 and 24.

75. 2.5 24. .8
A : C :: A : B below

Examp. 4. Given .00365, 6.57, .0425.

.00365 6.57 .0425 76.5
A : B :: A : B

2. *By the Prime Radius B.*

Examp. 1. Given 75, 4.5, and 2.5.

75. 4.5 2.5 .15
B : A :: B : A

Examp.

Examp. 2. Given 8.2, 24. and 1.5.

8.2 24. 1.5 4.4
B : A :: C : A below

See the N. B. in Multiplication *by the prime C.*

3. *By the Prime Radius C.*

Examp. 1. Given 15.4, 4.85 and 2.7.

15.4 4.85 2.7 .85
C : A :: C : A

Examp. 2. Given 15.4, 4.85 and 84.

15.4 4.85 84. 26.45
C : A :: B : A above

See N. B. as above.

To find the value of any vulgar fraction in parts of ſuch denomination into which its integer is, by the queſtion, ſuppoſed to be divided.

PROPORTION.

As the denominator of the given fraction
Is to the number of parts, into which its integer is ſuppoſed to be divided;
So is the numerator
To the number of parts.

Examp. 1. What is the value of $\frac{3}{4}$ of a pound ſterling, in ſhillings?

4 20. 3. 15. ſhillings
A : C :: A : C

Examp. 2. What is the value of $\frac{3}{5}$ of a pound ſterling, in pence?

5 240. 3. 144. pence
A : C :: A : C

E *Examp.*

Examp. 3. What is the value of $\frac{3}{4}$ of the hundred weight, in pounds averdupoise?

4 112. 3 84. pounds

A : C :: A : B

Note. The anſwers are all natural.

CHAP. IX.

Uſe of Diviſors on A, B, and C, of the Officer's Inſtrument in gauging of Areas.

TAB. I. II. and III.

PROPORTION.

AS the proper diviſor
Is to one of the given ſides;*
So is the other given ſide*
To the anſwer.

Note. If the area be a circle or ellipſis, for ſide* read *diameter.*

To find the number of places in the anſwer, *ſee* Rule of Three, *chap.* 8. *ſect.* 3.

SECT. I. *Of right lined Areas by the prime Radius A,* (Table I.)

Examples.

1. By diviſor MB:: (No. 1.)

Given a parallelogram, length 48 inches, breadth 30; what is its area in malt buſhels?

MB:: 48. 30. .67 anſwer

A : B :: A : B on

2. By

2. By divisor WSS: (No. 5.)

Given a parallelogram $18\frac{1}{2}$ inches by $13\frac{1}{2}$; what is its area in pounds of white soft soap?

WSS: 18.5 13.5 9.77 answer
A : C :: A : B below

3. By divisor Tpn: (No. 11.)

Given a parallelogram $8\frac{3}{4}$ inches by 6.25; what is its area in pounds of tallow neat?

Tpn: 8.75 6.25 1.74 answer
A : B :: A : C above

SECT. II. *Of Divisors on B and C, for circular and elliptical Areas.* (Tab. II. and III.)

N. B. As every part or point of the radius B or C, cannot be placed against every part or point of the radius A; therefore it is necessary that the abovesaid divisors be placed both on B and C. Hence observe,

If the prime of the proper divisor be $\left\{\begin{matrix}\text{greater}\\\text{less}\end{matrix}\right\}$ than the prime of either of the numbers which express the given dimensions, make $\left\{\begin{matrix}\text{B}\\\text{C}\end{matrix}\right\}$ prime radius.

Examples.

1. By divisor PG: (No. 14.)

Given a circle, diameter 18.6 inches; what is its area in pounds of plate glass?

PG: 18.6 18.6 29.3 answer
C : A :: C : A natural

2. By divisor ag.·. (No. 29.)

Given an ellipsis transverse diameter, $24\frac{1}{2}$ inches, conjugate 15; which is its area in ale gallons?

ag.·.	15.	24.5	1.02
B :	A ::	B :	A

Given the tranverse diameter, 75 inches, conjugate $44\frac{1}{2}$; what is its area?

ag.·.	75.	44.5	9.29
C :	A ::	C :	A

3. By divisor GS: (No. 30.)

Given an ellipsis, diameters 7.4 and 8.6; what is its area in pounds of green starch?

GS:	7.4	8.6	1.43 answer
C :	A ::	B :	A natural

N. B. All the above proportions may be performed by the inverted lines, as will be taught below.

CHAP.

CHAP. X.

Use of the Factors and Divisors on A, B, and C, of the Artificer's Instrument, in measuring of Superficies, &c.

TAB. VIII. and IX.

SECT. I. *Of Factors on lower B.* (Tab. VIII.)

PROPORTION.

AS unity on A,
Is to the proper factor on B;
So is the given length on A,
To the answer on B or C.

To find the number of places in the answer, *see* Multiplication, *chap.* VIII.

Examples.

1. By factor .Sic (No. 88.)

Given the circumference of a circle $25\frac{1}{2}$; what is the side of the greatest square which can be inscribed therein?

1		.Sic		25.5		5.73
A	:	B	::	A	:	B

2. By factor .Sec (No. 90.)

Given the circumference of a circle $48\frac{1}{2}$; what

 is

is the side of a square, whose area shall be equal to the area of the said circle?

1.		.Sec		48.5		13.65
A	:	B	::	A	:	C above

3. By factor .Sid (No. 91.)

Given the diameter of a circle 126; what is the side of the greatest square which can be inscribed therein?

1		.Sid		126.		89.1
A	:	B	::	A	:	B

SECT. II. *Of Divisors on A.* (Tab. IX.)

Proportion.

As the proper divisor on A,
Is to one of the given lengths on B or C;
So is the other given side on A,
To the answer on B or C.

To find the number of places in the answer, *see* Rule of Three, *chap.* VIII.

Examples.

1. In board measure.

By divisor 12: (No. 95.)

Given a board 25½ feet long, and 20 inches broad; what is its content in superficial feet?

12.		25.5		20.		42.5 answer
A	:	B	::	A	:	B natural

2. In

2. In land meaſure.

By diviſor L△: (N°. 96.)

Given a trapezium of land, whoſe diagonal is 8 chains, 45 links; and the ſum of the perpendiculars 8 chains and 16 links; what is its content in ſtatute acres?

L△:		8.45		8.16		3.44	
A	:	B	::	A	:	C	above

3. In ceiling, wainſcotting, painting, paving, &c.

By diviſor □Yd. (N°. 102.)

Given a ceiling, wainſcot, or pavement, length 25. feet, breadth 14.6; what is its contents in ſquare or ſuperficial yards?

□Yd.		25.		14.6		40.55	
A	:	C	::	A	:	B	below

4. In flooring, tiling, and roofing.

By diviſor □.·. (N°. 103.)

Given a piece of flooring, tiling, or roofing, length 235. feet, breadth 38.5; how many ſquares doth it contain?

□.·.		235.		38.5		90.4	
A	:	C	::	A	:	B	on natural

N. B. Becauſe the third number in the proportion is ſuppoſed to be found on the prime radius, you may ſuppoſe the prime 1. on A, to

 repreſent

represent the divisor; then make 235. the third number, and the answer will be also natural, viz.

□.·. 38.5	235.	90.4	
A : B	::	A ; B	

SECT. III. *Of Factors, which are also Divisors on A.*

Examples.

I. By factor and divisor Oc. (No. 97.)

1. By the factor.

Given the diameter of a circle 8.6; what is its circumference?

1	8.6	Oc.	27 answer
A :	B ::	A :	C natural

2. By the divisor.

Given the circumference of a circle 27.; what is its diameter?

Oc.	27.	1.	8.6 answer
A :	C ::	A :	B natural

II. By factor and divisor .rtd (No. 100.)

1. By the factor.

Given a piece of round timber, whose true content is found to be 31 feet; what is its content customary measure?

1.0	31.	.rtd	24.3
A :	C ::	A :	C natural

2. By

2. By the divisor.

Given a cylindrical piece of timber, whose content, by customary measure, is found to be 24.3 feet; what is its true content?

.rtd		24.3		1.0		31, answer
A	:	C	::	A	:	C

THE END OF THE FIRST PART.

A

K E Y

TO THE

MODERN SLIDING RULE.

P A R T II.

Of the inverted Line I *in the Rule of Three Inverse, and Compound Multiplication and Division; with the Use of the Divisors thereon in gauging and measuring Areas, Superficies, and Solids at* one Operation.

C H A P. I.

Of the Disposition of the Primes and Intermediates on the inverted Line; of compleating the Radii, and of working Proportions thereby.

SECT. I. *Of the Disposition of Primes and Intermediates on the inverted Line.*

THIS line consisteth of two like and equal radii, to the radius A, B, or C; but having the primes and intermediates thereon in an inverted order. *See* Description, *part* I. *chap.* I. Hence, with the slides B and C, the fourth pro-

proportional inverse to any three given numbers may be found, thus,

Set the first of the given numbers on the inverted line, to the second on B or C; then against the third on I, or I 2. is the proportional sought.

Let the given numbers be 6, 8, and 2.

Move the slides BC together till 6, on I stands right against 8 on B; then against 20, on I 2, is 2.4 on B, the fourth proportional.

SECT. II. *Of compleating the inverted Radii.*

This may be done without application of their parts, which are on the back side of the instrument.

Thus:

1. Place the inverted line even with the stock or rule, so that the prime 1. of the radius I 2, viz. the point marked 10. in the middle thereof, may stand near the middle of the instrument.

2. Place the slides B and C between the said inverted line and the radius A, so that they may join each other at the intermediate point 95 of B.

3. Move the slides B, C together, till the prime 1. of C, stand right against the prime 1. of radius I 2.

Now, it is easy to conceive, that the primes 1, 2, and 3, of the radius I, with their intermediates, doth each stand against the very same points of the radius C, against which the like primes 1, 2 and 3 of the radius I 2, with their inter-

intermediates, do each refpectively ftand on the radius B.

Again, the primes 4, 5, 6, 7, 8, and 9, of radius I 2, with their intermediates, do each ftand againft the very fame points of the radius B, againft which the like primes 4, 5, 6, 7, 8, and 9 of the radius I, with their intermediates, do each refpectively ftand of the radius C.

Hence the faid parts of the inverted radii do reprefent but one entire radius, viz. I or I 2.

From what hath been faid, it appears, 1. That when B is oblique, viz. when any part thereof ftands againft any part of radius I, if you fuppofe I to be prime radius, then will its other parts reprefent the radius C. Thus,

Move the flides together to the right, till the prime 1 of C, ftands againft the prime 6 of I; then all that part of radius B downwards, or to the left from prime 6, inclufive, (prime 1, of I 2, being its firft point) doth reprefent C.

2. When C is oblique, viz. when any part thereof ftands againft any part of I 2, if you fuppofe I 2 to be prime, then will its other part reprefent the radius B. Thus,

Move the flides together to the left, till prime 1 of C ftands againft prime 2 of I 2; then will the reft of the radius C upwards, or to the right from prime 2, exclufive, reprefent B.

And thus it will be in all pofitions of the flides B, C.

Hence,

Hence,

1. When B is oblique, that part thereof which ſtands againſt any part of I 2, becomes C.

2. When C is oblique, that part thereof which ſtands againſt any part of I, becomes B.

SECT. III. *Of finding the Number of Places in the fourth Proportional, and of rectifying the Inſtrument.*

It is evident from inſpection of the inſtrument, that if the third number be greater than the firſt, the fourth will be leſs than the ſecond: alſo, if the third number be leſs than the firſt, the fourth will be greater than the ſecond. Thus,

Set 1.5 on C to 4 on I; then againſt 3 on I is 2 on C.

Here the third number 3 is leſs than the firſt 4, and the fourth, viz. 2, is greater than 1.5 the ſecond.

Now, if you ſuppoſe the third number to become 30, viz. one place more than 3, the ſaid 30 muſt be ſuppoſed to be found on the next radius above I, viz. I 2; conſequently the fourth proportional will be found on B, viz. the next radius below the collateral; therefore it will have one place leſs than in the former caſe. Hence,

To find the number of places in the fourth proportional:

RULE.

RULE.

If the anſwer falls on the collateral, it will conſiſt of as many places $\left\{\begin{matrix}\text{leſs}\\ \text{more}\end{matrix}\right\}$ than the ſecond number, as the third hath $\left\{\begin{matrix}\text{more}\\ \text{leſs}\end{matrix}\right\}$ than the firſt.

N. B. If the anſwer falls $\left\{\begin{matrix}\text{above}\\ \text{below}\end{matrix}\right\}$ the collateral, it will have one place $\left\{\begin{matrix}\text{more}\\ \text{leſs}\end{matrix}\right\}$ than if it falls on

Examp. 1. Given the numbers 4, 15, and 20; what is the fourth proportional?

4.	15.	20	3.
I :	C ::	I2 :	C on

Examp. 2. Given the numbers 15, 36, and 9.

15.	36.	6.	90.
I2 :	C ::	I :	B below

Examp. 3. Given 120, 13, and 6.

120.	13.	6.	260.
I2 :	C ::	I :	B below

Examp. 4. Given 8, 70, and 20.

8.	70.	20.	28.
I :	B ::	I2 :	C above

Examp. 5. Given 60. .3 and 1.5.

60.	.3	1.5	12.
I :	B ::	I2 :	C above

N. B. When the difference of places in the firſt and third number $=1$. and the leaſt of them

is found on I, the anſwer will be natural, as in the 1ſt, 2d, and 4th of the above examples.

But the fourth proportional may be more expeditiouſly and obviouſly found by the following method.

Move the inverted line to the left, till the braſs pin in the middle thereof, doth ſtand againſt the braſs pin on the left hand of upper A, and compleat the radius I.

Then is the inſtrument rectified for this purpoſe.

And if $\left\{\begin{matrix}B\\C\end{matrix}\right\}$ be collateral, the anſwer will fall on or $\left\{\begin{matrix}\text{above}\\\text{below}\end{matrix}\right\}$ it.

Examp. 1. Given 6, 40, and 50, to find the fourth proportional.

6.	40.	50.	4.8
I :	B ::	I :	B

The anſwer falls on collateral; therefore it hath as many places leſs than the ſecond number, as the third hath more than the firſt.

Examp. 2. Given 6, 7, and 20.

6.	7.	20.	2.1
I :	B ::	I :	C

The third number hath one place more than the firſt; therefore, if the anſwer had fallen on the collateral, it would have had one place leſs than the ſecond number; but it falls above it, and conſequently hath one place more; viz. equal places with the ſecond.

Examp.

Examp. 3. Given 3.4, 15. and 12.

3.4		15.		12		4.25.
I	:	C	::	I	:	C

Anſwer falls on, therefore it hath as many places leſs than the ſecond, as the third hath more than the firſt.

Examp. 4. Given 14. 2.5, and 5.

14.		2.5		5.		7.
I	:	C	::	I	:	B

If the anſwer had fallen on, it would have had one place more than the ſecond number, becauſe the third hath one leſs than the firſt; but it falls below, therefore it hath equal place therewith.

CHAP. II.

Of Multiplication, Simple and Compound, by the inverted Lines.

SECT. I. *Of Simple Multiplication, or how to find the product of any two Numbers multiplied into each other.*

Lemma 1.

IF any four numbers are in geometrical proportion inverſe, the product of the firſt and ſecond numbers will be equal to the product of the other two.

Thus, in the last example of the foregoing chapter, where the proportionals are 14, 2.5, 5, and 7.

$14.\times 2.5 = 5.\times 7. = 35.$

Hence, if the two given factors be made the first and second terms in the proportion, and unity or 1, the third; the fourth proportional inverse thereto, will be the product of the said factors. Compare with the direct rule, part 1.

Examp. Let the given factors be 4 and 2.

Rectify as taught in the foregoing chapter, then place 2 on C to 4 on I; and against 1 on I, is 8, the fourth proportional on C. That is,

$$\begin{array}{ccccccc} 4 & & 2. & & 1 & & 8. \\ I & : & C & :: & I & : & C \end{array}$$

Here $4\times 2 = 1.\times 8. = 8$ the product.

Again, let the given factors be 6 and 7.

Place 6 on B to 7 on I; and against 1 on I is 42 on C.

$$\text{That is } \begin{array}{ccccccc} 7. & & 6 & & 1. & & 42. \\ I & : & B & :: & I & : & C \end{array}$$

Here $7.\times 6 = 1.\times 42. = 42$ the product.

To find the number of places in the product.

Because unity or 1, is always the third number in the proportion, it follows, that

If $\left\{\begin{array}{c} C \\ B \end{array}\right\}$ be collateral, the answer will fall $\left\{\begin{array}{c} \text{on} \\ \text{above} \end{array}\right\}$ it.

Hence,

Hence,

If $\left\{ \begin{matrix} C \\ B \end{matrix} \right\}$ be collateral, the number of places in the product will be $\left\{ \begin{matrix} \text{one leſs than} \\ \text{equal to} \end{matrix} \right\}$ the ſum of the number of places in both factors. *See part* 1. *chap.* VIII.

SECT. II. *Of Compound Multiplication, or how to find the Product of three Numbers, when multiplied continually into each other, at one operation.*

Lemma 2.

In all poſitions of the ſlides, whatever prime or intermediate of the inverted radius I, ſtands againſt any prime, or intermediate of the radius C; the like prime or intermediate with the former on I 2, doth ſtand againſt the like prime or intermediate with the latter on the radius B. Thus,

Rectify as above taught, (*See laſt chap. ſect.* 3.) then place the prime 3 on C, to the prime 2, on I; then againſt the prime 2, on I 2, is the prime 3 on B; and at the ſame time, as prime 1, on I doth ſtand againſt prime 6, on C; ſo doth prime 1, on I 2, ſtand againſt prime 6, on B.

Hence,

The product of any two numbers will be always found againſt the prime 1, of radius I 2.

F 2 Thus,

Thus, in the laſt example.

4.		2.		1.		8
I	:	C	::	I2	:	B

Here, C is collateral; therefore the product hath one place leſs than the ſum of the number of places in both the factors. *See the laſt ſection.*

Again,

6.		7		1.		42.
I	:	B	::	I2	:	B

Here, B is collateral; therefore the number of places in the product, equal the ſum of the number of places in both factors. *See as above.*

Now, the braſs pin G, at the left hand of upupper A, doth ſtand right againſt the prime 1, of lower radius A, therefore in this poſition of the inverted line, the product of any two numbers will be always found againſt the prime 1, of radius A, conſequently the product of any two numbers may be multiplied by a third number, at one ſet of the inſtrument, by the Rule of Multiplication by the direct Lines.

Examp. Given the number 5. 6. and 3.

	5.		6.		1.		30. $=5.\times6.$
It will be 1ſt	I	:	B	::	I2	:	B.

and in the ſame poſition,

	1.		30		3		90. $=30\times3$
It will be 2dly	A	:	B	::	A	:	B

that is, at one operation,

5.		6.		3.		90.
I	:	B	::	A	:	B

Hence,

Hence, observe the lower edge of B, and also of C, when used with the inverted line, is to be esteemed the same radius with its respective upper edge.

Consequently,

If you place either of any three given factors on I, to either of the other two on B or C, against the third on A, you will have the compound product of the said three numbers.

Of finding the number of places in the product of any three given numbers.

Seeing, the product of any two numbers is always found against the prime 1, of radius A, it follows, that,

If the third factor consisteth of one place of integers, and the second product be found on B, it will consist of the same number of places with the first product; therefore,

If $\left\{ \begin{matrix} C \\ B \end{matrix} \right\}$ be the collateral, and the second product falls on B, it will consist of $\left\{ \begin{matrix} \text{two places} \\ \text{one place} \end{matrix} \right\}$ less, than the sum of the number of places in all the factors. *See preceding chapter.*

Now, if you suppose the third factor to be increased or decreased by any number of places, the product must be supposed to be increased or decreased respectively by the same number of places. *See Corol. part* 1. *chap.* III. *sect.* 3.

Consequently, the number of places in the second product will bear the same proportion to

the ſum of the number of places, as in the former caſe; hence,

1. If $\left\{\begin{matrix}C\\B\end{matrix}\right\}$ be collateral, and the anſwer falls on B, it will conſiſt of $\left\{\begin{matrix}\text{two places}\\\text{one place}\end{matrix}\right\}$ leſs than the ſum of the number of places in all the factors.

2. If the anſwer falls on C, it will have one place more in each caſe: hence the *General Rule*,

To find the number of places in the product of any three numbers.

If the anſwer falls $\left\{\begin{matrix}\text{above}\\\text{on}\\\text{below}\end{matrix}\right\}$ the collateral, it will conſiſt of $\left\{\begin{matrix}\text{as many places as}\\\text{one place}\\\text{two places}\end{matrix}\right.\left.\begin{matrix}\\ \text{less than}\\ \end{matrix}\right\}$ the ſum of the number of places in all the factors.

Examp. 1. Given 15. 2.4 and 12. to find the product.

15. 2.4 12. 432.
I : C :: A : B

The ſum of the number of places $=5$ anſwer, falls below, therefore hath 3 places.

Examp. 2. Given 2.4, 1.5 and 50.

2.4 1.5 50 180
I : C :: A : C

Anſwer falls on, and ſum of the number of places $=4$.

Examp. 3. Given 3.5, 7.6 and 1.5

3.5 7.6 1.5 39.9
I : B :: A : B

Anſwer

Anſwer falls on, and ſum of the number of places = 3.

Examp. 4. Given 6, .75 and 50.

6. .75 50. 225.

I : B :: A : C

Anſwer falls above the collateral, and the ſum of the number of places = 3, the ſecond number being a fraction of the firſt order.·. Anſwer hath three places.

CHAP. III.

Of Diviſion, and the Rule of Three Direct, by the inverted Line.

SECT. I. *Of Diviſion.*

N. B. IF unity or 1, be made the firſt term in the proportion, the dividend the ſecond, and the diviſor the third; the fourth proportional inverſe thereto, will be the quotient ariſing from ſuch diviſion. *See* Diviſion, *part* 1. *chap.* VI. *alſo* Lemma 1. *of laſt chapter.*

Thus, let it be required to find the quotient of 40 divided by 2.

Rectify as taught, *chap.* I. *ſect.* 3.

Then ſet 40 on C to 1 on I; and againſt 2 on I is 20 on C. Anſwer falls on.

Again, let the quotient of 40 by 8 be required.

Set 40 on C to 1 on I, and against 8 on I is 5 on B. Answer falls below.

SECT. II. *Of the Rule of Three Direct, by the inverted Line, or how to find the Quotient arising from the Product of any two Factors or Numbers divided by a third Number.*

Lemma 1.

In every four geometrical proportional direct, the product of the two means or middle numbers will be equal to the product of the two extremes.

Hence, by *Lemma* 1. *of the preceeding chapter.*

If the two means of any four proportional direct, be made the first and second terms, and the first term or divisor be made the third; the fourth proportional inverse thereto, will be the quotient arising from the product of the said two means, divided by the said divisor. Thus,

Let the given factors be 4 and 15, and divisor 3.

Set the inverted line even with the stock or rule: then set 15 on C, to 4 on I; and against 3 on I 2, is 20 on B, the quotient sought.

To find when the answer falls on or off the collateral, &c.

Let the radii I and I 2, be supposed to be completed, and

Let the divisor or third number in the proportion, be always found on I 2, and the answer on B. Then,

From

From what hath been faid in the firft chapter of this part, it follows, that

1. If $\left\{\begin{matrix}1\\12\end{matrix}\right\}$ be prime, and the fecond number found on $\left\{\begin{matrix}C\\B\end{matrix}\right\}$ the anfwer falls on the collateral.

2. If $\left\{\begin{matrix}1\\12\end{matrix}\right\}$ be prime, and the fecond number be found on $\left\{\begin{matrix}B\\C\end{matrix}\right\}$ the anfwer falls $\left\{\begin{matrix}\text{above}\\\text{below}\end{matrix}\right\}$ the collateral.

Now let $\left\{\begin{matrix}C\\B\end{matrix}\right\}$ be efteemed the *natural* collateral of $\left\{\begin{matrix}1\\12\end{matrix}\right\}$ then may be known when the anfwer falls on or off the collateral, by the following

RULE.

1. If the fecond number in the proportion be found on a *natural* collateral, the anfwer falls on.

2. If the fecond number be found on $\left\{\begin{matrix}B\\C\end{matrix}\right\}$ *unnatural*, the anfwer falls $\left\{\begin{matrix}\text{above.}\\\text{below.}\end{matrix}\right\}$

SECT. III. *Of finding the Number of Places in the Anfwer.*

It is evident, that if the three given agents in any proportion, confift each of one integral place, the fourth proportional thereto, will confift of one integral place alfo, if it be found on the collateral. *See part* 1. *chap*. III.

It

It is evident also, that if either of the two factors, be supposed to be increased or decreased by any number of places, the fourth proportional will be increased or decreased respectively by the like number of places. *See part* 1. *chap.* III.

But it is also obvious, that in either case, the difference between the number of places in the divisor, and the sum of the number of places in the two factors, will be increased or decreased respectively, in the same proportion. Or,

If the divisor be supposed to be increased or decreased, by any number of places, the difference between the number of places in the divisor, and the sum of the number of places in the dividend, will be increased or decreased by the like number of places respectively. Consequently,

If the answer falls on the collateral, the number of places therein, will be equal to the difference of the number of places in the divisor, and the sum of the number of places in the two factors. And consequently,

If the answer falls $\left\{\begin{matrix}\text{above}\\ \text{below}\end{matrix}\right\}$ the collateral, it will have one place $\left\{\begin{matrix}\text{more}\\ \text{less}\end{matrix}\right\}$ than the said difference. Hence the

RULE.

To find the number of places in the answer.

1. If the collateral be B or C *natural*, the number of places in the answer, will be equal to the difference between the number of places in the divisor, and the sum of the number of places in the

the two factors, viz. the first and second number in the proportion.

2. If the collateral be $\left\{ \begin{matrix} B \\ C \end{matrix} \right\}$ *unnatural*, the number of places in the answer will be one $\left\{ \begin{matrix} \text{more} \\ \text{less} \end{matrix} \right\}$ than the abovesaid difference. *See the Rule, sect.* 2.

Examples.

1. By the prime I.

Examp. 1. Given the factors 5 and 15, and the divisor 3, to find the quotient.

$$\begin{array}{ccccccc} 5 & & 15. & & 3 & & 25.\ \text{answer} \\ \text{I} & : & \text{C} & :: & \text{I2} & : & \text{B} \end{array}$$

The second number is found on a *natural* collateral; therefore the number of places in the answer, will be equal to the difference of places in the divisor, and the two factors. By the difference of places in the divisor, and the two factors, is here and hereafter meant, the difference between the number of places in the divisor, and the sum of the number of places in both factors.

Examp. 2. Given the factors 6 and 75, divisor 15.

$$\begin{array}{ccccccc} 6. & & 75. & & 15 & & 30.\ \text{answer} \\ \text{I} & : & \text{B} & :: & \text{I2} & : & \text{B} \end{array}$$

The second number is found on B *unnatural*; therefore the number of places in the answer will be one more than the difference of places.

2. By

2. By the prime I 2.

Examp. 1. Given the factors 1.5 and 75, divisor 25.

1.5 75 25 4.5 answer
I2 : B :: I2 : B

The second number is found on a *natural* collateral; therefore the number of places in the answer, will be equal to the difference of places.

Examp. 2. Given the factors 12 and 14, divisor 4.

12 14. 4. 42. answer
I2 : C :: I2 : B

The second number is found on C *unnatural*; therefore the answer hath one place less than the difference of places.

CHAP. IV.

Of the Rule of Three Compound, by the inverted Line, or how to find the Quotient arising from the Divisor of the Product of a continued Multiplication of three Numbers into each other, by any given Divisor, at one Operation.

N. B. IF two numbers be multiplied into each other, and their product be divided by a third number; if the quotient arising therefrom be multiplied by a fourth number, this last product will be the same, as if the three given factors

tors had been multiplied continually into each other, and their product divided by the ſame diviſor. Thus,

Let the given factors be 4, 15 and 3, and diviſor 2.

I ſay $4 \times 15 \div 2 \times 3 = 4 \times 15 . \times 3 \div 2$

Thus:

$4. \times 15. = 60.$	and	$4. \times 15. = 60.$
$60. \div 2. = 30.$		$60. \times 3. = 180.$
$30. \times 3. = 90.$		$180. \div 2. = 90.$

Hence, having by the *Rule given in laſt chapter*, found the quotient ariſing from the diviſion of the product of either two of the three given factors, by the given diviſor; move all the ſlides together, till the ſaid quotient ſtands right againſt the prime 1, of the radius A.

Then will the diviſor ſtand right againſt the point marked G, at the left hand of upper A; ſo will the ſaid quotient be fitted for its multiplication by any third factor, by the Rule of Multiplication, by the direct lines.

N. B. Hence all areas and ſuperficies will be found on B, againſt prime 1 of radius A: and

The ſolidity of any priſm, &c. will be found on B or C, againſt its length, breadth, or depth, on radius A.

Given, as above, the factors 4, 15 and 3, and the diviſor 2.

4 15. 2 30=quotient

1. Set I to C then againſt I2 is B

Now move all the ſlides together till 30 on lower B, ſtand right againſt 1 on A; then will the

the divisor 2, stand right against the brass pin G, on upper A.

Then will it be

1. 90. 3. 90. = answer
A : B :: A : B

That is,

4 15. 3 90 the answer
I : C :: A : B

Hence,

To rectify the instrument for finding the quotient arising from the division of the product of any three numbers into each other, divided by a fourth number.

R U L E.

Place any prime or intermediate on B, right against the prime 1 of the radius A; then place the given divisor on I 2, right against the abovesaid prime or intermediate respectively.

Then will the answer be found by the following

P R O P O R T I O N.

As either of the three given factors on I or I 2,
Is to either of the other two on B or C;
So is the third on A,
To the quotient on B or C.

SECT. II. *Of finding the Number of Places in the Answer.*

Seeing when only two factors are concerned, if the collateral be B or C natural, and the answer

 falls

falls on B, it will consist of equal places with the difference of places in the divisor, and the said factors: And

That if the third factor consist of one integral place, and the second product be found on B, it will consist of equal places with the above quotient. It follows,

That the number of places in the second product, if found on B, will be one less than the difference (*See* Rule, *chap.* III.) of places in the divisor, and the three factors. Hence,

To find the number of places in the answer.

GENERAL RULE.

1. If the collateral be B or C *natural*, the answer, if it falls on B, will consist of one place less than the difference of places in the divisor, and the three factors.

2. If the collateral be $\left\{\begin{matrix}\text{B}\\\text{C}\end{matrix}\right\}$ *unnatural*, the answer, if it falls on B, will consist of $\left\{\begin{matrix}\text{equal places with}\\\text{2 places less than}\end{matrix}\right\}$ the abovesaid difference.

3. If the answer falls on C, it will consist of one place more than, if it falls on B, in every case.

Examples.

1. By the prime I.

Examp. 1. Given the factors 4, 15 and 3, and divisor 2.

Rectify to the divisor as above taught.

4 15 3 90

Then it will be I : C :: A : B

The

The collateral is *natural*, and the anſwer found on B, therefore it hath 1 place leſs than the difference of places in the diviſor, and thoſe in all the factors.

Factors = 4. places, diviſor = 1. ∴ diff. = 3.

Note. If the third factor had been 4, the anſwer would have been 120. *See the Rule above.*

Examp. 2. Given the factors 5, 8 and 3, and diviſor 2.

5		8		4		80
I	:	B	::	A	:	B

Or,

8		5		4		80
I	:	B	::	A	:	B

The ſecond number is found on B *unnatural*; therefore the number of places in the anſwer will be equal the difference of places.

If the third number had been 7, the anſwer would have fallen on C, and would have been 140.

2. By the prime I 2.

Rectify to the diviſor 2. Then

Examp. 1. Given the factors 15, 4 and 3, and diviſor 2.

15		4		3		90
I2	:	B	::	A	:	B

The collateral is *natural*, and the anſwer found on B; therefore it will have 1 place leſs than the difference of places.

Compare with the firſt example by prime I, and *Lemma* 2. *chap.* II.

Examp.

Examp. 2. Given the factors 13, 1.2 and 12, and divisor 2.

13	1.2	12	93.6
I2 :	C ::	A :	B

Or,

1.2	13	12	93.6
I2 :	C ::	A :	B

The collateral is C unnatural, and the anſwer falls on B. The difference of places =4.·. The anſwer hath 2 places leſs than the ſaid difference.

Note. If the third number had been 20, the anſwer would have fallen on C, and would have been 156, viz. one place more.

N. B. The like is to be obſerved by any other diviſor.

CHAP. V.

Of the fixed Diviſors on the inverted Line, and of rectifying the Inſtrument for any Purpoſe.

SECT. I. *Of the Diviſors on the upper Edge of Radius* I 2.

THESE are each put exactly againſt that point of its lower edge, which expreſſeth its reſpective number.

Thus, the diviſor MB:: of the officer's inſtrument, (No. 25 in the table) is put exactly againſt that point of the radius I 2, which repreſents the number 2738, viz. the proper diviſor for circular or elliptical meaſure of malt buſhels.

Alſo, the diviſor ag.·. (No. 29) is put exactly againſt that point which repreſents the number 359, viz. the proper diviſor for circular and elliptical meaſure of ale gallons; and ſo of the reſt of the diviſors on I 2.

Hence,

If either of the diviſors on the radius I 2, be placed right againſt the braſs pin G, at the left hand of upper A; then is the inſtrument rectified to the ſaid diviſor.

SECT. II. *Of the Diviſors on the upper Edge of the Radius* I.

Theſe are each put exactly againſt that point of the lower edge, which expreſſeth its reſpective number.

Thus, the diviſor WSS:. on the officer's inſtrument, (No. 18.) is put exactly againſt that point of the radius I, which repreſents the number 32.54, viz. the proper diviſor for circular or elliptical meaſure of ſoft ſoap.

Alſo, the diviſor Tp; (No. 20.) is placed exactly againſt that point of the radius I, which repreſents the number 38.55, the proper diviſor for circular or elliptical meaſure of tallow neat.

Hence,

If either of the diviſors on the radius I be placed right againſt the braſs pin G, at the right hand of upper A, then, becauſe the radii I, and I 2, are repreſentatives of each other, the ſaid

diviſor

divifor on I 2, will ftand againft the point G, of the left hand of upper A; and fo will the inftrument be rectified to the faid divifor. The like is to be obferved of the reft of the divifors on I.

SECT. III. *Of the Divifors on upper* A.

Obferve, in all pofitions of the inverted line, whatever prime or intermediate of the radius I 2 doth ftand againft the prime 1, of the radius A; the like prime or intermediate on A, doth ftand againft the prime 1, or brafs pin, on the radius I 2.

Thus, place the prime 4 on I 2, right againft the prime 1 of the radius A, (*fee the* Rule *to* rectify, *chap.* IV.) then will the prime 4 on A, ftand right againft the brafs pin, or prime 1 of the radius I 2. *See chap.* I. *fect.* 1.

Again, move the inverted line to the right, till the prime 5, of the radius I 2, ftands right againft the point G, or prime 1, of the radius A; then will the like prime 5 on A, ftand right againft the prime 1, or brafs pin of the radius I 2.

Now, the divifors on upper A, are each placed right againft its refpective proper point of the line A.

Hence,

If you place the prime 1 on brafs pin on upper edge of the radius I 2, right againft either of the divifors, on upper A, then will the faid divifor on I 2, ftand right againft the prime 1, of the radius

A, and consequently the instrument will be rectified to the said divisor.

The like is to be observed of any other divisor on upper A.

CHAP. VI.

Of Compound Multiplication and Rule of Three Inverse.

To rectify the Instrument.

RULE.

PLACE the prime 1, viz. the brass pin in the middle of the inverted line, right against the brass pin marked G, at the left hand of upper A, and compleat the radius I; (*see chap.* I. *sect.* 2.) then

SECT. I. *Compound Multiplication will be performed by the following*

PROPORTION.

As one of the factors on I,
Is to either of the others on B or C;
So is the third on A,
To the product on B or C.

See chap. II. *Lemma* 1, &c.

To

To find the number of places in the product.

RULE.

If the anſwer falls $\left\{\begin{array}{l}\text{above}\\ \text{on}\\ \text{below}\end{array}\right\}$ the *collateral*, (*ſee chap*. II. *ſect*. 1.) it will conſiſt of $\left\{\begin{array}{l}\text{as many places as}\\ \left.\begin{array}{l}\text{one place}\\ \text{two places}\end{array}\right\}\text{leſs than}\end{array}\right\}$ the ſum of the number of places in all the factors. *See chap*. II. *ſect*. 1.

N. B. The lower edge of B, and alſo of C, is to be eſteemed the ſame radius as its reſpective upper edge.

Examp. 1. Multiply 5, 7 and 8, into each other.

5.	7.	8.	280.	anſwer
I :	B ::	A :	C	above

Examp. 2. Multiply 40, 8 and 2.

40.	8.	2.	640.	anſwer
I :	B ::	A :	B	on

Examp. 3. Multiply 2.4, 2.5 and 5.

2.4	2.5	5.	30	anſwer
I :	C ::	A :	C	on

Examp. 4. Multiply .3, 1.2 and 20.

.3	1.2	20	7.2	anſwer
I :	C ::	A :	B	below

SECT. II. *Rule of Three Inverse.*

Rectify as above; then will the fourth proportional inverse be found by the following

PROPORTION.

As the first number on I,
Is to the second on B or C;
So is the third on I,
To the fourth on B or C.

See chap. I. *sect.* 1.

To find the number of places in the answer.

RULE.

As many places as the third number hath $\left\{\begin{array}{l}\text{more}\\\text{less}\end{array}\right\}$ than the first; so many will the answer have $\left\{\begin{array}{l}\text{less}\\\text{more}\end{array}\right\}$ than the second, if it falls on the *collateral.*

If it fall $\left\{\begin{array}{l}\text{above}\\\text{below}\end{array}\right\}$ $\left\{\begin{array}{l}\text{add}\\\text{deduct}\end{array}\right\}$ one place. *See chap.* I. *sect.* 3.

Examp. 1. Given 8, 16 and 40.

8	16	40	3.2	answer
I :	B ::	I :	B on	

Examp. 2. Given 6, 50 and 20.

6.	50.	20.	15.	answer
I :	B ::	I :	C above	

Examp. 3. Given 5, 12 and 20.

5.	12.	20.	13.	answer
I :	C ::	I :	C	on

Examp. 4. Given 15, 24 and 8.

15.	24.	8.	45.	answer
I :	C ::	I :	B	below

CHAP. VII.

How to rectify the Instrument for gauging and measuring Areas, Superficies and Salids; and how to find the Number of Places in the Answers, &c.

SECT. I. *To rectify the Instrument.*

RULES.

1. For divisors on upper A.

PLACE the brass pin in the middle of the inverted line, (viz. the prime 1 of the radius I 2) right against the proper divisor on upper A. *See chap.* V. *sect.* 3.

2. For divisors on radius I.

Place the proper divisor thereon, right against the brass pin at the right hand of the line upper A. *See chap.* V. *sect.* 2.

3. For divisors on radius I 2.

Place the proper divisor thereon, right against the brass pin at the left hand of upper A. *See chap.* V. *sect.* 1.

Having rectified the instrument by one or other of the above rules, as the case in hand may require, compleat the radius I, or I 2. Then

The superficial content or area of the base of any rectangular, circular, or elliptical prism, and also the solidity of the said prism, may be found at one operation, by the following

GENERAL PROPORTIONS.

As one of the given sides* of the base on I or I 2,
Is to the other on B or C;
So is unity on A,
To the area of the base on B or C.

And,

So is the length of the prism on A,
To its solidity or content on B or C.

See Proportion, *chap.* IV. *sect.* 1.

If the base be circular or elliptical, for sides*, read *diameters.*

N. B. All areas or superficies are found right against unity on A; consequently will always fall on B. *See N. B. also* Rule, *chap.* IV. *sect.* 1.

N. A. It matters not which of the three given dimensions be made the first, second or third number in the proportion, except when the area be required, or content at any several given depths. *See* Proportion, *chap.* IV.

SECT.

SECT. II. *To find the Number of Places in the Answer.*

N. B. Let the radius B be esteemed the natural collateral of the radius I 2; and C the natural collateral of the radius I.

Then will the number of places in any answer be found by the following

GENERAL RULE.

$$\text{If the collateral be} \left\{\begin{array}{l}\text{B, unnatural}\\ \text{B or C, natural}\\ \text{C, unnatural}\end{array}\right\} \text{and the}$$

answer be found on B, it will consist of

$$\left\{\begin{array}{l}\text{as many places as}\\ \left.\begin{array}{l}\text{one place}\\ \text{two places}\end{array}\right\}\text{less than}\end{array}\right\} \text{the difference of places}$$

in the divisor, and the sum of the number of places in all the factors. *See chap.* IV.

N. B. If the number of places in the divisor, equal or exceed the sum of the number of places in all the factors, and the answer falls on B; it will be expressed by a fraction having as many cyphers prefixed as the said difference is.

N. A. If the answer falls on C, it will in all cases consist of one place more than if it had fallen on B.

SECT. III. *To find the Sum of the Number of Places in any three given Numbers.*

I. If all the given numbers are integral or mixed, or one or two of them fractions of the first order, then,

The sum of the number of places in them all, will be equal to the sum of the number of integral places in all of them.

II. If one or two of the given numbers are fractions of any other order, then

1. If the number of integral places exceed the number of cyphers prefixed in the fractions, the said excess will be the sum of the number of places in them all.

2. If the number of cyphers prefixed, be equal to the number of integral places; the sum of the number of places will be negative, and will be expressed by a fraction of the first order.

3. If the sum of the number of cyphers prefixed, exceed the number of integral places, the sum of the number of places will be expressed by a fraction, having as many cyphers prefixed, as the said excess is.

III. If all the given numbers are fractions, the sum of the number of places in all, will be expressed by a fraction, with as many cyphers prefixed, as the sum of the number of cyphers prefixed in all.

CHAP.

CHAP. VIII.

Use of Divisors on the inverted Line of the Officer's *Instrument, in gauging of Areas and Solids, at* one Operation.

SECT. I. *Of Divisors on upper* A, *for rectilineal Bases.* (Tab. I.)

Examples.

1. By divisor MB:: (No. 1.)

Examp. 1. GIVEN a *parallelepipid*, base 64 inches, by 18, depth 45: what is its area and content in *malt* bushels?

Rectify to the proper divisor, (*see chap.* VII.) and compleat radius I. Then *see* General Proportions, *in preceding chap.*

64 18 1 .535 area 45 24.1 content
I : C :: A : B :: A : C

The second number in the proportion is found on a natural collateral. The sum of the number places in all the factors for the $\left\{\begin{array}{l}\text{area}\\ \text{content}\end{array}\right\} = \left\{\begin{array}{c}5\\ 6\end{array}\right\}$ and the number of places in the divisor $=4$; therefore the difference of places $= \left\{\begin{array}{c}1\\ 2\end{array}\right\}$ consequently, the number expressing the $\left\{\begin{array}{l}\text{area}\\ \text{content}\end{array}\right\}$ will $\left\{\begin{array}{l}\text{be a fraction of the first order,}\\ \text{consist of two integral places.}\end{array}\right\}$ *See* General Rule, *chap.* VII. *sect.* 2.

N. B.

N. B. As the ſlides now ſtand, you have the content at every inch deep. Thus,

$$\text{againſt}\left\{\begin{matrix}2,\\3,\\4,\\5,\end{matrix}\right\}\begin{matrix}\text{on}\\\text{A,}\\\text{is}\end{matrix}\left\{\begin{matrix}1.07\\1.6\\2.14\\2.67\end{matrix}\right\}\begin{matrix}\text{the con-}\\\text{tent at}\end{matrix}\left\{\begin{matrix}2,\\3,\\4,\\5,\end{matrix}\right\}\begin{matrix}\text{inches}\\\text{deep.}\end{matrix}$$

Examp. 2. Given the baſe 13 inches by 54, depth 26.

13		54.		1		.326 area		26.		8.48 content
I2	:	B	::	A	:	B	::	A	:	B

The ſecond number is on a natural collateral.

N. B. If the breadth 18 in the firſt example, had been made the firſt number in the proportion, it would have been found on I 2, and the length 64, on its natural collateral B; and ſo the anſwer would have come out as above. *See Lemma* 2. *chap.* 2. The like is to be obſerved of the ſecond example.

Examp. 3. Given the baſe 89.5 by 75.4, depth 6.5.

89.5		75.4		1		3.13 ar.		6.5		20.39 cont.
I	:	B	::	A	:	B	::	A	:	C

The ſecond number is found on B *unnatural.*

Examp. 4. Given the baſe 154. by 126, depth $36\frac{1}{2}$.

154.		126		1		9. ar.		36.5		329. cont.
I2	:	C	::	A	:	B	::	A	:	C

The ſecond number on C *unnatural.*

2. By

2. By divisor HS: (N°. 8.)

Examp. 1. Given a *parallelopepid*, base 22.5 by 4.75, depth 17.3; what is its area and content in pounds of *hard soap?*

Rectify and compleat the radius I. Then

22.5	4.75	1	3.93 ar.	17.3	68.1 cont:
I2 :	B ::	A :	B ::	A :	B

Examp. 2. Given a cube, each side 9 inches.

9	9	1	2.98	9	26.8
I :	B ::	A :	B ::	A :	C

Examp. 3. Given a *parallelopepid*, base 15.6 by 12.8, depth 35.½.

15.6	12.8	1.	7.35	35.5	261.
I2 :	C ::	A :	B ::	A :	C

3. By divisors on DS: (N°. 13.)

Examp. 1. Given a *parallelopepid*, base 6.2 by 32.5 inches, depth 46.4; what is its area and content in pounds of *dry starch?*

Rectify, to the proper divisor, and compleat radius I 2. Then

32.5	6.2	1.	5.	46.4	232.
I2 :	B ::	A :	B ::	A :	C

Examp. 2. Given the base, each side 8¾, depth 32.½.

8.75	8.75	1	1.9	32.5	61.7
I :	B ::	A :	B ::	A :	B

Examp.

Examp. 3. Given the base 12 inches by 18, depth .75.

18		12		1		5.35		.75		4.01
12	:	C	::	A	:	B	::	A	:	C

Examp. 4. Given the base, each side ¼ of an inch, depth 8.4.

.75		.75		1.		.014		8.4		.117
I	:	B	::	A	:	B	::	A	:	C

See chap. VII. *sect.* 3. *also part* 1. *chap.* VIII. *sect.* 2.

Note. The like is to be observed of the rest of the divisors on upper A.

N. A. There will be no necessity of completing the radius, except when the proper divisor is found on one of the slides.

SECT. II. *By divisors on* I *and* I 2, *for elliptical and circular Bases.* (Tab. II. and III.)

N. B. There is no other difference in the operations by these and the former, than in rectifying the instrument, which *see the preceding chapter.*

Examples.

1. By divisor Tpn: (No. 21.)

Examp. 1. Given a *parallelopepid* base 9½ inches by 17.6, depth 25; what is its area and content in *tallow pounds* neat?

Rectify as taught in the foregoing chapter, and compleat the radius I 2. Then

9.5

9.5 17.6 1 4.2 25. 104.
I : C :: A : B :: A : C

Examp. 2. Given the base 18 inches by 12, depth 15.

18 12 1 5.4 15. 81.
I2 : C :: A : B :: A : B

2. By divisor ag. (No. 29.)

Examp. 1. Given a *cylindroid*, whose tranverse diameter of base is 98 inches, conjugate 56½, depth 15; what is its area and content in *ale gallons?*

Rectify and complete the radius I 2. Then

98 56.5 1. 15.5 15. 231.
I : B :: A : B :: A : C

Examp. 2. Given a cylinder, diameter 25, depth 64.

25. 25. 1. 1.74 64. 111.
I2 : B :: A : B :: A : C

Note. The like is to be observed of the rest of the divisors on I and I 2.

CHAP.

CHAP. IX.

Use of Divisors on upper A of the inverted Line of the Artificer's *Instrument, in measuring of Superficies and Solids at* one Operation.

TAB. X.

SECT. I. *Of Board and Timber Measure; and also of Sawer's Work.*

Examples.

1. Of board measure.

By divisor Bst: (No. 105.)

Examp. 1. GIVEN a stock of boards, length 26 feet, breadth 14 inches, number 15; how many feet of boards doth the said stock contain?

Rectify to the divisor, and complete the radius I. *See chap.* VII. Then

26. 14 15. 455. answer
I : C :: A : B

The second number is found on a natural collateral; difference of places =4. Answer falls below. *See chap.* VII.

Note. In this situation of the slides, you have the content of one single board, or of a stock of any number of boards, of the same dimensions.

Thus,

Thus,

$$\text{against}\left\{\begin{array}{l}1,\\2,\\3,\\4,\\5,\end{array}\right\}\begin{array}{l}\text{on A}\\\text{you}\\\text{have}\end{array}\left\{\begin{array}{c}30.33\\60.66\\90.9\\121.3\\151.6\end{array}\right\}\begin{array}{l}\text{the}\\\text{con-}\\\text{tent}\\\text{of}\end{array}\left\{\begin{array}{c}1\\2\\3\\4\\5\end{array}\right\}\text{boards, \&c.}$$

Hence obſerve, always make the number of boards the *third* number in the proportion.

Examp. 2. Given a *ſtock* of 8 boards, length $6\frac{1}{2}$ feet, breadth $9\frac{1}{4}$ inches.

6.5		9.25		1.		5.=1.brd. 8				40. ſtock
I	:	B	::	A	:	B	::	A	:	C

The *ſecond* number is found on B *unnatural.*

Examp. 3. Given a ſtock of 9 boards, 11 feet by $10\frac{1}{2}$ inches.

11.		10.5		1		9.62=1.brd. 9				86.6 ſtock
12	:	C	::	A	:	B	::	A	:	C

The *ſecond* number is found on C *unnatural.*

2. In rectangled *timber* meaſure.

By *diviſor* □Tim.·. (No. 109.)

Examp. 1. Given a *parallelopepid*; baſe $8\frac{1}{2}$ inches by 14.8, length $36\frac{1}{2}$ feet; what is the content of one foot of length thereof, and alſo of the whole prism?

Rectify to the proper *diviſor*, &c. Then

8.5		14.8		1		.873		36.5		31.8
I	:	C	::	A	:	B	::	A	:	C

Examp. 2. Given a parallelopepid, base 8.7 inches by 9.5, length 14.6 feet.

9.5		8.7		1		.574		14.6		8.37
I	:	B	::	A	:	B	::	A	:	B

Examp. 3. Given the base 12.6 by 10.8, length 26.4.

12.6		10.8		1		.945		26.4		24.9
I2	:	C	::	A	:	B	::	A	:	C

N. B. Always make the length the *third* term in the proportion.

3. To measure a *set* of like and equal *joists* of any number.

RULE.

Take the *breadth*, and also the *depth* or *thickness* of one of the *joists*, in inches and decimal parts, and its *length* in feet and decimal parts, and multiply the *breadth* by the number of *joists* in the set; then will the *content* of the whole set be found by the following

PROPORTION.

As the *depth* of the joist on I or I2,
Is to its *length* on B or C;
So is the sum of all the *breadths* on A,
To the *content* of the set on B or C.

Examp. 1. Given a set of 8 *joists*, $3\frac{1}{2}$ inches by $6\frac{1}{4}$; *length* $12\frac{1}{2}$ *feet*; how many *feet* of timber is contained in the said set?

$8 \times 3.5 = 28$

6.25		12.5		28		15.2	ans.
I	:	C	::	A	:	C	

The *second* number is found on a *natural* collatural.

Examp.

Examp. 2. Given a ſet of 12 *joiſts*, 8 inches by 3, *length* 9 feet.

12×3=36	8	9	36	18 anſwer
	I	: B ::	A :	C

The *ſecond* number is found on B *unnatural*.

Examp. 3. Given 24 *joiſts*, 1.05 inches by 1.25, *length* 10½ feet.

	1,	24	1.05	25.2
firſt	A :	B ::	A :	B

Then

1.25	10.5	25.2	2.39
I2 :	C ::	A :	C

The *ſecond* number is found on C *unnatural*.

Note. If ſeveral ſets of *joiſts* are of *equal girt*, and each ſet of *different lengths*; make the lengths the *third* numbers in the proportion, and the *content* of each ſet will be *found* at one operation.

4. By *diviſor* ⊖Tim.·. (No. 115.)

Examp. 1. Given a cylindroid or elliptical priſm, *tranverſe* diameter of its baſe 4¼ inches, *conjugate* 1½, length .95 of a foot; what is the content of *one foot* of its length, and alſo of the *priſm*?

Rectify to the proper *diviſor*, &c. Then

1.5	4.25	1	.0347	.95	.033
I2 :	B ::	A :	B ::	A :	C

Examp. 2. Given a cylindroid, *baſe* 8.6 inches by 42, *length* 12½ feet.

8.6	42.	1	1.97	12.5	24.6
I :	B ::	A :	B ::	A :	B

Examp. 3. Given a cylinder, *diameter* of baſe 12.8 inches, *length* 46 feet.

12.8		12.8		1		.89		46.		41.
I2	:	C	::	A	:	B	::	A	:	C

5. In *ſawer* work.

By *diviſor* K:: (N°. 111.)

Examp. 1. Given a *ſtock* of *boards*, *length* 14 feet, *breadth* 26 inches, *kerfs* 21; how many *hundreds* of ſawing doth the ſaid *ſtock* contain?

Rectify to the proper *diviſor*, &c. Then

14		26		21.		5.3 anſwer
I2	:	B	::	A	:	B

Examp. 2. Given a *ſtock* of 7 kerfs, $9\frac{3}{4}$ inches by $34\frac{1}{2}$ feet.

9.75		34.5		7.		1.63
I	:	B	::	A	:	C

Examp. 3. Given a *ſtock* of 8 kerfs, 11 inches by 12.8 feet.

12.8		11		8		.782
I2	:	C	::	A	:	C

Examp. 4. Given a *ſtock* 39 kerfs, length $44\frac{3}{4}$ feet, by $28\frac{1}{2}$ inches.

44.75		28.5		39.		34.5
I	:	C	::	A	:	C

SECT.

SECT. II. *Of Brickwork.*

How to find the number of *rods* or *poles*, and alſo the number of *ſtatute bricks* in any walling, at any given *thickneſs*.

1. For the number of rods or poles.

By *diviſor* BW.·. (N°. 116.)

Examp. 1. Given a brick wall, *length* 156 feet, *height* 9.4; how many *rods* of walling doth it contain at *ſtandard* thickneſs of $1\frac{1}{2}$ brick?

Rectify to the proper *diviſor*, &c. Then

9.4		156		1.5		5.38 anſwer
I	:	C	::	A	:	B

Note. In this ſituation of the *ſlides*, you have the number of *rods* in a wall of any *thickneſs*, being of the ſame *length* and *height* with the above. Thus,

$$\text{againſt}\left\{\begin{array}{l}1,\\2,\\2.5\\3,\\4,\end{array}\right\}\text{bricks thick, is}\left\{\begin{array}{r}3.59\\7.18\\8.97\\10.77\\14.36\end{array}\right\}\text{rods, \&c.}$$

Examp. 2. Given a wall $62\frac{1}{2}$ feet *long*, $8\frac{1}{2}$ *high*; what is its *content* at 1.5, alſo at $4\frac{1}{2}$ *bricks* thick?

8.5		62.5		1.5		1.95 ſtand.		4.5		5.85
I	:	B	::	A	:	B	::	A	:	B

Examp. 3. Given a wall 245 *feet* by 14.6, $3\frac{1}{2}$ *bricks* thick.

245.		14.6		1.5		13.14		3.5		30.6
I2	:	C	::	A	:	C	::	A	:	C

2. For the *number* of bricks in any walling.

1. By *divisor* Brk°: (No. 118.)

Examp. 1. How many *thousand* of *statute* bricks are required to build a wall 85 feet *long*, and 9 *high*, at *standard* thickness of $1\frac{1}{2}$ brick?

Rectify to the proper *divisor*, &c. Then

9.		85.		1.5		12.2
12	:	B	::	A	:	B

Answer 12,200.

N. B. Allowance is here made for cement or mortar.

Examp. 2. How many *bricks* are required to build a wall of 45 *feet* by 78 at $1\frac{1}{2}$, also at $2\frac{1}{2}$ *bricks* thick?

45.		78.		1.5		56. stand.		2.5		93.6
12	:	B	::	A	:	B	::	A	:	B

Answer at 1.5 br. 56000 at 2.5. 93600.

Examp. 3. How many *bricks* are required for a wall of $28\frac{1}{2}$ *feet* by 15.6, at $1\frac{1}{2}$ and $4\frac{1}{2}$ *bricks*?

28.5		15.6		1.5		7.1		4.5		21.4
12	:	C	::	A	:	B	::	A	:	C

Answer at $1\frac{1}{2}$ br. 7100 at $4\frac{1}{2}$, 21400.

Examp. 4. How many *bricks* are required for a wall $9\frac{1}{2}$ feet *high*, 6 *long*, at 6 bricks.

9.5		6		6.		3.65
1	:	C	::	A	:	C

Answer 3650.

N. B. For finding the number of *places* in the anſwers, &c. *See* General Rules, *chap.* VII.

SECT. III. *Of Glazier's Work.*

N. B. This admits of two caſes, viz. when the *height* is taken in feet and decimal parts, and the *breadth* in inches and decimal parts; and, when *both* dimenſions are taken in inches and parts.

Examples.

1. When the height or length is given in *feet* and parts, and the breadth in *inches* and parts.

By the *diviſor* GL: (No. 106.)

Examp. 1. Given 24 lights, each $3\frac{1}{2}$ *feet* by $22\frac{1}{2}$ *inches*; how many *feet* of glazing is contained in *each* light, and alſo in *all* of them?

Rectify to the proper *diviſor*, &c. Then

3.5	22.5	1	6.56=1.l.	24.	157. total
I :	C ::	A :	B ::	A :	C

Examp. 2. Given 14 lights, each $3\frac{1}{4}$ *feet* by $11\frac{1}{2}$ *inches.*

11.5	3.25	1	3.11	14.	43.6
12 :	B ::	A :	B ::	A :	B

Examp. 3. Given 32 lights, each 9.9 *inches* by 1.58 *feet.*

9.9	1.58	1.	1.3	32.	41.7
I :	B ::	A :	B ::	A :	B

2. When the *length* and *breadth* are both given in *inches* and decimal parts.

By *divisor* Gl.·. (No. 110.)

Examp. 1. Given 4 lights, each 128 *inches* by 62.

Rectify to the proper *divisor*, &c. Then

128.	62.	1.	55.	4.	220.
I2 :	B ::	A :	B ::	A :	C

Examp. 2. Given 6 lights, each $44\frac{1}{2}$ *inches* by $28\frac{1}{2}$.

44.5	28.5	1.	8.8	6.	52.8
I :	C ::	A :	B ::	A :	C

Examp. 3. Given 4 lights, each 56 *inches* by $36\frac{1}{2}$.

36.5	56.	1.	14.2	4.	56.7
I :	B ::	A :	B ::	A :	B

See General Proportions, *and* Rules *to find the number of places in the answers, chap.* VII. *also the* first example, *chap.* VIII.

N. B. If the height and breadth are both taken in *feet* and decimal parts, rectify as for *Compound Multiplication*, and let C be esteemed the *natural collateral* of I, and proceed as above.

SECT.

SECT. IV. *Of Gauging of Ships.*

1. By *divisor* Shw. (No. 104.)

Examp. 1. Given a ship of war, *length* 150 feet, *breadth* 39, *depth* $19\frac{1}{2}$; what is her *burthen* in tons?

Rectify to the proper *divisor*. Then

150. 39. 19.5 1140 answer.
I : C :: A : C

Examp. 2. Given the *length* 95 feet, *breadth* 38, *depth* 14.

95. 38. 14. 505. answer
I : B :: A : B

Examp. 3. Given the *length* 128 feet, *breadth* 32, *depth* 16.

128. 32. 16. 655. answer
I : C :: A : B

2. By *divisor* Shm: (No. 120.)

Examp. 1. Given a vessel, *length* 96 feet, *breadth* 38, *depth* 14; what is its content or burthen, as a *merchant* man?

Rectify to the *divisor*, &c. Then

96. 38. 14. 536.
I : C :: A : B

Examp.

Examp. 2. Given the *length* 87 feet, *breadth* 22, *depth* 15.

87.		22.		15.		302. answer	
I2	:	B	::	A	:	B	

Examp. 3. Given the *length* 150 feet, *breadth* 39, *depth* $19\frac{1}{2}$.

150.		39.		19.5		1200.
I2	:	C	::	A	:	C

3. By *divisor* Shft: (No. 119.)

Examp. 1. Given a vessel, *length* 87 feet, *breadth* 22, *depth* 15; what is its content or burthen *statute*?

Rectify to the proper *divisor*. Then

87.		22.		15.		305.
I2	:	B	::	A	:	B

Examp. 2. Given the *length* 98 feet, *breadth* 26, *depth* 14.

98.		26.		14.		379.
I	:	C	::	A	:	B

Examp. 3. Given the *length* 126 feet, *breadth* $42\frac{1}{2}$, *depth* $24\frac{1}{2}$.

126.		42.5		24.5		1395.
I2	:	C	::	A	:	C

See General Proportions, *and* Rules *to find the number of places in the answers, chap.* VII. *See also the* first example, *chap.* VIII.

THE END OF THE SECOND PART.

A KEY

A
KEY
TO THE
MODERN SLIDING RULE.

PART III.

Description of the Line D, *with its Use in extracting the Roots of* Squares, *and in the* Proportions *of like Areas and Superficies: Also in* gauging *and* measuring *of Areas, Superficies and Solids.*

CHAP. I.

Description of the Line D, *and of the Slides* B *and* C, *when used therewith; also of the imaginary* Radii *above and below* D.

SECT. I. *Description of the Line* D.

THIS line is put on the *opposite* side or plane of the instrument to the line A. It consisteth of one single *radius* of numbers divided into two equal parts, at the *intermediate* point 3162, &c. The one part being placed *above*, the other *below* the slides; each part being exactly equal in length to the *radius* B or C.

SECT.

SECT. II. *Of the Slides* B *and* C.

Theſe, when taken together, may eaſily be conceived to repreſent *four* diſtinct radii of numbers; each edge of each *ſlide* repreſenting a *radius*; nevertheleſs, when uſed with the line D, they repreſent *three* entire and diſtinct radii only, one above another. Thus,

1. Place the *ſlides* B and C, between the two parts of the *radius* D, and move them together, till the prime 1 on D, doth ſtand right againſt the prime 1 of the *upper* edge of B; then will the prime 1 of the *upper* edge of C, ſtand right againſt the *intermediate* point 3162, &c. of the *upper* edge of D. This I call the *direct* poſition of the radius B.

Now the *lower* edge of D, may be conceived to be joined to its upper edge at the point 3162, &c. aforeſaid, and conſequently to ſtand againſt the *upper* edge of C, in the very ſame manner as it now doth againſt the *lower* edge of B; that is, the primes 4, 5, 6, 7, 8, and 9, with their *intermediates*, may now be ſuppoſed to ſtand againſt the ſame *primes* and *intermediates* of the *upper* edge of C, as they really do on the *lower* edge of B.

Therefore the *lower* edge of B, doth repreſent the *upper* edge of C, and may be eſteemed the ſame *radius*.

Hence obſerve, the *lower* edge of B doth repreſent the next radius *above* its own *upper* edge, which therefore I call B 2.

2. Move

2. Move the *ſlides* B and C together to the *left*, till the point 10 of C, ſtands right againſt the point 10 on the *lower* edge of D; then will the *prime* 1 of the *lower* edge of C, ſtand right againſt the *intermediate* point 3162, &c. of the *lower* part of D. This I call the *direct* poſition of the radius C.

Now the *upper* edge of D, with its *prime* 1, 2, 3, with their *intermediates*, may be ſuppoſed to be joined to its *lower* edge at the aforeſaid point 3162, &c. and ſtanding againſt the *lower* edge of B, as it doth againſt the *upper* edge of C.

Therefore the *upper* edge of C, doth repreſent the *lower* edge of B, and may be eſteemed the ſame radius.

Now, it is obvious, the *lower* edge of C, is the next radius *above* the *lower* edge of B; therefore,

The *lower* edge of C, doth repreſent the next radius *above* its own *upper* edge; which therefore I call C 2.

Hence obſerve,

The *ſlides* B and C, when uſed with the radius D, do in every *oblique* poſition, repreſent *three* entire and diſtinct *radii*,

$$\text{viz.}\ \left\{\begin{array}{l}\text{1ſt,}\\ \text{2d,}\\ \text{3d,}\end{array}\right\}\ \begin{array}{c}\text{the ra-}\\ \text{dius}\end{array}\left\{\begin{array}{l}\text{B,}\\ \text{C and B 2,}\\ \text{C 2,}\end{array}\right\}\ \text{or}\ \left\{\begin{array}{l}\text{lower}\\ \text{middle}\\ \text{upper}\end{array}\right\}\ \begin{array}{c}\text{ra-}\\ \text{dius.}\end{array}$$

SECT.

SECT. III. *Of the imaginary Radii above and below* D.

1. *Of the Radius above* D.

Place B *direct*; now you are to ſuppoſe another *radius* like unto D, running *upwards* from its firſt point 10 on D, towards the *right* hand, with the *primes* 1, 2 and 3, and their *intermediates* ſtanding or abutting againſt the *lower* edge of C, viz. C 2, in the ſame manner as the like *primes* 1, 2 and 3, with their *intermediates* on D, do againſt the *primes* and *intermediates* of the *upper* edge of B.

So that the *radius* C 2, is now ſuppoſed to ſtand againſt the *radius* next *above* D, but is repreſented by the *radius* B, ſtanding againſt the like part of the *radius* D.

2. *Of the Radii below* D.

Place C 2 *direct*; now you muſt imagine a like *radius* to D, running down from the point or *prime* 1 of D, towards the *left*, and having thereon the *primes* 9, 8, 7, 6, 5 and 4, with their *intermediates*, ſtanding againſt the like *primes* and *intermediates* reſpectively, of the *upper* edge of B, as the like *primes* 9, 8, 7, 6, &c. with their *intermediates* on D, doth againſt thoſe of the *lower* edge of C or C 2.

So that the *radius* B, is now ſuppoſed to ſtand againſt the *radius* next *below* D; but is repreſented by the *radius* C 2, or *lower* edge of C.

Note. All other *imiginary* radii *above* and *below* B, C and D, may be reduced to the *radii* B, C and D, in the ſame manner as the radii *above* and *below* the radii A, B and C, are reduced thereto. *See* Corollary, *part* I. *chap.* III. *ſect.* 3.

CHAP. II.

Of the Diſpoſition *of Primes and Intermediates on the* Radius D; *and how to find the* Radius *whereon any Number is repreſented.*

SECT. I. *Of the Diſpoſitions of the Primes and Intermediates.*

THE *primes* and *intermediates* on theſe lines, are diſpoſed in ſuch manner, as, that when either the *radius* B or C, is in *direct* poſition, it becomes with the *radius* D, a table of *ſquares* with their *roots.*

Thus,

Place C *direct*; now have you all *ſquares* repreſented on C, with their reſpective *roots* on D.

Thus, againſt the *ſquare* 4 on C, you have its *root* 2 on D, and againſt the *ſquare* 9 on C, is its *root* 3 on D; alſo, againſt the *ſquare* 25 on C 2, is its *root* 5 on D, and againſt the *ſquare* 64, is the *root* 8, &c.

SECT.

SECT. II. *To find the Radius whereon any Square is to be sought.*

1. For *integral* or *mixed* numbers.

Place C *direct*, and let the *prime* 1 of D, and also of C, represent *unity*.

Then will all *squares* or numbers, consisting of an *odd* number of *integral* places, be found on C, and all *squares* or numbers, consisting of an *even* number of places, on C 2. *See sect. 1.*

2. For *fractional* squares.

Place C *direct*, and let the point 10 on D, and also on C, represent *unity* or 1.

Then will all *fractions* of the *first* order, also all *fractions* having an *even* number of cyphers prefixed, be represented on C 2; and all *fractions* which have an *odd* number of cyphers prefixed, will be found on C. *See as above.*

SECT. III. *To find the Number of Places in the Root of any given Square.*

1. Of *integral* or *mixed* squares.

It is evident from what hath been said, that if the given number consisteth of *two* integral places, its *root* will consist of *one* integral place, viz. one *half* the number of places in the given *square*.

Now seeing the *radius* D is equal in *length*, to *both* radii B and C, it follows, that for every *two* places

places, any given ſquare is ſuppoſed to be *encreaſed* or *decreaſed*, the *root* of ſuch ſquare muſt be ſuppoſed to be *encreaſed* or *decreaſed* reſpectively, by one place. Compare with *Corollary*, *part* I. *chap*. III. *ſect*. 3.

Hence,

1. If the number of *integral* places in any given ſquare be *even*, the number of places in its root will be *half* that number. Again,

Seeing the root of any ſquare conſiſting of one *integral* place, doth conſiſt of *equal* places with the root of a ſquare of *two* integral places;

Hence,

2. If the number of *integral* places in the given ſquare be *odd*, add one thereto; and *half* their ſum will be the number of places in its root.

2. Of *fractional* ſquares.

From what hath been ſaid in the foregoing ſection, it appears, that the *root* of a fraction of the *firſt* order, will be a fraction of the *firſt* order.

It alſo appears, that if the given fraction be of the *ſecond* order, its root will be a fraction of the *firſt* order.

Hence,

If the given *ſquare* be of the *firſt* or *ſecond* order, its *root* will be of the *firſt* order alſo.

When the given *ſquare* is of any other *order* then,

1. If the number of *cyphers* prefixed be *even*, half that number muſt be *prefixed* in its root.

2. If the number of cyphers are *odd*; deduct one, and half the *remainder* must be prefixed in the *root*.

CHAP. III.

Of Proportions by the Line D.

FROM what hath been said (*chap.* II. *sect.* 1.) it appears, that the *radius* D, is a line of *roots*, and that each *prime* and *intermediate* thereon, doth represent its own *square*.

Thus, the prime 2 doth represent its *square* 4; the prime 3, its *square* 9; and the prime 4, its *square* 16.

Also, the intermediate 2.5, doth represent its *square* 6.25, and the intermediate 8.5, its *square* 72.25, &c.

Hence,

If D be *prime* radius, the fourth proportional found by these lines, will bear the same *proportion* to the *second* number, as the *square* of the third, doth to the *square* of the first. Thus,

$$\begin{array}{ccccccc} 2 & & 8 & & 4 & & 32 \\ D & : & B & :: & D & : & B2 \end{array}$$

That is,

$$\begin{array}{ccccccc} 4 & & 8 & & 16. & & 32 \\ A & : & B & :: & A & : & B \end{array}$$

Now, if the *area* or superficial content of the *base* of any prism, be multiplied into the *length* of the said prism, and the product thereof be divided

vided by a proper *divisor*, the quotient hereof, will be the *solidity* of the said prism, in that *measure*, to which the said *divisor* is adapted. Thus in the last example,

If 4 be the proper *divisor*, 8 the length of the *prism*, and the *area* of its base 16, its *content* will be 32.

Which by the line D will run thus:

As the *square* root of the divisor on D, is to the *length* of the prism on B or C; so is the *square* root of the *area* of its base on B, to its *content* on B or C. *See* Example 1.

Hence the *square* root of the *divisor* for any purpose, is the proper *gauge point* for the same purpose. Hence it will be,

As the proper *gauge point* on D, is to the *length* of any prism on B or C; so is the *square* root of the *area* of its base on D, to its *solidity* or content on B or C.

Lemma.

The *solidity* of all prisms of *equal height* or *length*, having similar or like *bases*, are to each other, as are the *squares* of their *homologous*, or *like* sides of their bases. Hence

The *solidity* of all prisms will be found on the instrument by the following

PROPORTION.

As the proper *gauge point* on D,
  Is to the *length* of the given prism on B or C;
So is the side of its *base* on D,
  To its *solidity* on B or C.

CHAP. IV.

Use of the Gauge Points *on the Line* D *of the* Officer's *Instrument, in gauging of the* circular *Areas and Prisms.*

TAB. VII.

SECT. I. *Of circular* Areas.

PROPORTION.

AS the proper *gauge point* on D,
  Is to *unity* or 1 on C or C 2,*
So is the given *diameter* on D,†
  To the *area* on B or C.

* *N.B. Unity* or 1, neither multiplies nor divides.

† *See* Lemma *in the foregoing chapter.*

Observe, from what hath been said in the former chapter, the *middle radius* C and B 2, is in all *oblique* positions of the slides, within the compass of the line D.

N. A. Unity being the *second* number in the proportion, C or C 2, will be always *collateral.*

Hence,

Hence,

The anſwer, in regard to the *collateral*, may

fall $\left\{\begin{array}{l}\text{thereon}\\ \text{on the firſt above or below it.}\\ \text{on the ſecond below it.}\end{array}\right.$

That is,

If C be *collateral*, it may fall $\left\{\begin{array}{l}\text{thereon.}\\ \text{on the next above it.}\\ \text{or on the next below it.}\end{array}\right.$

If C 2 be *collateral*, it may fall $\left\{\begin{array}{l}\text{thereon.}\\ \text{on the next below it.}\\ \text{or on the ſecond below it.}\end{array}\right.$

See chap. I. *ſect.* 2 *and* 3.

To find the number of places in the anſwer.

Becauſe *radius* D is equal in length to the *two* radii B and C, (*ſee chap.* I. *ſect.* 1.) hence the

GENERAL RULE.

As many places as the *third* number in the proportion hath $\left\{\begin{array}{l}\text{more}\\ \text{leſs}\end{array}\right\}$ than the *firſt*; *twice* ſo many places will the anſwer have $\left\{\begin{array}{l}\text{more}\\ \text{leſs}\end{array}\right\}$ than the *ſecond*, if it falls on the *collateral*. *See chap* I. *ſect.* 2. *alſo* Corollary, *chap.* III. *part* I.

If the anſwer falls on the $\left\{\begin{array}{l}\text{firſt}\\ \text{ſecond}\end{array}\right\}$ radius *above* the *collateral*, add $\left\{\begin{array}{l}\text{one place.}\\ \text{two places,}\end{array}\right.$

If it falls on the $\left\{\begin{array}{l}\text{firſt}\\ \text{ſecond}\end{array}\right\}$ radius *below*, deduct $\left\{\begin{array}{l}\text{one place.}\\ \text{two places.}\end{array}\right.$

Note. When the *third* number in the proportion consists of *equal* places with the *first*, the answer will be *natural.*

Examples.

1. By *gauge point* WG: (No. 72.)

Examp. 1. Given a *circle*, diameter 25.4 inches; what is its area in *wine gallons?*

WG:	1	25.4	2.19 answer
D :	C ::	D :	C on

Examp. 2. Given the diameter 36 inches.

WG:	1	36.	4.4 answer
D :	C ::	D :	B2 on

Examp. 3. Given the diameter $75\frac{1}{2}$.

WG:	1	75.5	19.38 answer
D :	C ::	D :	C2 first above

Examp. 4. Given the diameter 15.6.

WG:	1	15.6	.827 answer
D :	C ::	D :	B first below

The above *answers* are all *natural.*

Examp. 5. Given the diameter 218.

WG:	1	218	161.5 answer
D :	C ::	D :	C

The *third* number hath 1 place *more* than the *first*, and the answer falls on the *collateral*; therefore it hath 2 places *more* than the *second* number, viz. more than 1.

Examp.

Examp. 6. Given the diameter 9.5.

WG:		1.		9.5		.306 answer
D	:	C	::	D	:	C2

The *third* number hath one place *less* than the *first*, therefore if the *answer* had fallen on the *collateral*, it would have had 2 places *less* than the *second*; but it falls on the next radius *above* it, therefore it hath 1 place *less* than the *second* number.

Or thus.

Because *radius* B and C 2 represent each other, (*see chap.* I. *sect.* 3.) suppose the *third number* to be found on the next *radius below* D, then will the *answer* be found *natural.*

That is,

WG:		1.		9.5		.306
D	:	C	::	—D	:	B

Examp. 7. Given the diameter 126.

WG:		1.		126.		53.97
D	:	C	::	D	:	B.

The *third* number hath 1 place *more* than the *first*, therefore if the *answer* had fallen on the *collateral*, it would have had 2 places *more* than the *second*; but it falls on the next radius *below*, and consequently hath 1 place *more*.

Or thus,

Suppose the *third number* to be found on the next *radius above* D, then will the *answer* be found *natural.* (*See as above.*)

Thus,

W.G.		1.		126		53.97
D	:	C	::	+D	:	C2

N. B. The like may be done in the following *Examples*, which are marked with an *Asterism*.

Examp. 8. Given the diameter 350:

WG:	1.	350.	415.
D :	C ::	D :	B2

The *third* number hath 1 place *more* than the *first*, and the *answer* falls on the *collateral*, therefore it hath 2 places *more* than the *second* number.

2. By *gauge point* HS. (No. 83.)

* *Examp.* 1. Given the diameter .864 inches; what is its area in *pounds* of *hard soap*?

HS.	1.	.864	.0216
D :	C ::	D :	C2

The *third* number hath 1 place *less* than the *first*; answer falls *on*.

Examp. 2. Given the diameter 34½.

HS.	.1	34.5	34.4
D :	C2 ::	D :	B2

The *third* number hath 1 place *more* than the *first*; answer falls on next *below*.

Examp. 3. Given the diameter 31.5.

HS.	1.	31.5	28.7
D :	C ::	D :	C

This is the same case with the last.

* *Examp.* 4. Given the diameter 146.

HS.	1.	146.	616.9
D :	C ::	D :	B

The *third* number hath 2 places *more* than the *firſt*, therefore, if the *anſwer* had fallen on the *collateral*, it would have had 4 places *more* than the *ſecond*; but it falls *on* the *ſecond* below, therefore it hath 2 places *more*.

SECT. II. *For circular* Priſms.

PROPORTION.

As the *gauge point* point on D,
Is to the *depth* of the cylinder on B or C,
So is the *diameter* on D,
To the *content* on B or C.
See Lemma, *chap.* III.

Note.

1. If B be *collateral*, the anſwer may fall { thereon / on the firſt / or on the ſecond } radius *above* the *collateral*.

2. If C 2 be *collateral*, the anſwer may fall { thereon / on the firſt / or on the ſecond } radius *below* the *collateral*.

3. If the middle radius be *collateral*, the anſwer may fall { on / on firſt *above* / on firſt *below* } the *collateral*.

Examples.

1. By *gauge point* AG: (N°. 75.)

By *collateral* B.

* *Examp.* 1. Given a cylinder, *diameter* 120 inches, *depth* .75; what is its content in *ale gallons*?

AG:	.75	120.	30.07 anſwer
D :	B ::	D :	B

The

The *third* number hath 1 place *more* than the *firſt*; anſwer falls on the *collateral*, therefore it hath 2 places *more* than the *ſecond* number.

Examp. 2. Given a cylinder, *depth* .075, *diameter* 236.

AG: .075 236. 11.63 anſwer

D : B :: D : C firſt *above*

The *third* number hath one place *more* than the *firſt*; anſwer falls *above*, and ∴ hath three places *more* than the *ſecond* number.

Examp. 3. Given *depth* 7.5, *diameter* 6.25.

AG: 7.5 6.25 .815 anſwer

D : B :: D : B2 firſt *above*

* *Examp.* 4. Given depth 75, diameter 9.8.

AG: 75. 9.8 20.05 anſwer

D : B :: D : C2 ſecond *above*

By *collateral* C.

Examp. 1. Given the depth 12.6, diameter 2.5.

AG: 12.6 2.5 .219 anſwer

D : C :: D : C *on*

Examp. 2. Given depth 12.6, diameter 45½.

AG: 126. 45.5 72.6 anſwer

D : C :: D : B2 on *natural*

* *Examp.* 3. Given depth 126. diameter 8.71.

AG: 126. 8.75 26.8 anſwer

D : C :: D : C2 firſt *above*

* *Examp.* 4. Given depth 1.26, diameter 144.

AG: 1.26 144. 72.74 anſwer

D : C :: D : B firſt *below*

2. By

2. By *gauge point* DS. (No. 87.)

By *collateral* B 2.

Examp. 1. Given a cylinder depth 8.4 inches, diameter 38.5; what is its content in *pounds* of *dry ſtarch*?

DS.	8.4		38.5		242.7 answer	
D	: B2	::	D	:	B2 *on*	

Examp. 2. Given the depth .84, diameter 27.

DS.	.84		27.		12.2 anſwer
D	: B2	::	D	:	C *on*

* *Examp.* 3. Given the depth 84, diameter .995.

DS.	84.		.995		1.62 anſwer
D	: B2	::	D	:	C2 firſt *above*

Examp. 4. Given the depth .84, diameter 124.

DS.	.84		124.		251. anſwer
D	: B2	:	D	:	B firſt *below*

By *collateral* C 2.

Examp. 1. Given the depth 28.5, diameter .75.

DS.	28.5		.75		.3125 anſwer
D	: C2	::	D	:	C2 *on*

Examp. 2. Given the depth 2.85, diameter 41.5.

DS.	2.85		41.5		95.7 anſwer
D	: C2	::	D	:	B2 firſt *below*

* *Examp.* 3. Given the depth 2.85, diameter 15.

DS.	2.85		15.		12.5
D	: C2	::	D	:	C firſt *below*

* *Examp.* 4. Given the depth .285, diameter 105.

DS.	.285		105.		61.25 anſwer
D	: C2	::	D	:	B ſecond *below*

CHAP.

CHAP. V.

Of gauging and ullaging *of Casks.*

SECT. I. *Of* gauging *of Casks.*

THE *concavity* or *capacity* of every *cask*, is supposed to be in the form of one or other *of the* four following *solids*, viz.

the middle frustum of

1. A spheriod.
2. A parabolic spindle.
3. Two parabolical connoids abutting against 1 com. base.
4. Two cones abutting against 1 common base.

Now, in order to ascertain the true *capacity* or content of either of these casks by the *instrument*, it must first be reduced to a *cylinder* of equal capacity.

For this purpose, there are on the backside of one of the *slides* or *sliders*, three lines, called lines of *varieties*, abutting against a line of *inches*, and and marked *spheriod*, 2d *variety*, and 3d *variety*, which are used in the following manner;

Subtract the *head diameter* of the cask to be gauged, from its *bung diameter*, and observe the difference; which difference seek on the line of *inches*, and against it, on the proper line of *variety*, you will find a *number*, which being added to the *head* diameter, the *sum* will be the *mean* required.

Examples.

Examples.

		1ſt var.	2d var.	3d var.
Given	bung	36.	45.	32.
	head	27.	38.	29.
	difference=	9.	7.	3.

Then,

On var. $\left\{\begin{matrix}1,\\2,\\3,\end{matrix}\right\}$ againſt $\left\{\begin{matrix}9,\\7,\\3,\end{matrix}\right\}$ inches, is $\left\{\begin{matrix}6.3\\4.45\\1.68\end{matrix}\right.$

Therefore,

	1ſt var.	2d var.	3d var.
To *head* diameter	27.	38.	29.
add	6.3	4.45	1.68
The means=	33.3	42.45	30.68

Thus, any *caſk* being reduced to a *cylinder*, its *content* in ale or wine gallons, &c. will be found on the *inſtrument* by the following

PROPORTION.

As the proper *gauge point* on D,
Is to the *length* of the caſk on B or C;
So is the *mean diameter* on D,
To its *content* on B or C.

See Proportion, *chap.* III.

Examp. 1. Given a caſk of the *firſt* variety, whoſe *bung* diameter is 34 inches, and *head* 27, and its *length* 43; what is its content in *ale* and *wine gallons?*

34.—27=7. difference

Then againſt 7 inches, on the line of *varieties*, (ſperiod) is 4.9 $\therefore$ 27+4.9 =31.9 is the *mean* diameter.

Therefore

Therefore it will be

AG:	- - -	- - -	122. ale	} gallons	
WG:	43.	31.9	148. wine		
D :	B ::	D :	C		

Examp. 2. Given a caſk of the *ſecond* variety, *bung* 36 inches, *head* 32, length 48.

$36-32=4$.

Againſt 4 inches, on *variety* 2, is 2.52 ∴32+2.52=34.52, the *mean* diameter. Therefore

AG:	- - -	- - -	160. ale	} gallons
WG:	48.	34.52	197. wine	
D :	B ::	D :	B2	

Note. The like is to be obſerved in the *third* variety.

SECT. II. *Of the Lines of Segments in* ullaging *of Caſks.*

Theſe lines are put on the edges or narrower planes of the inſtrument. That immediately under the line A, is for caſks *lying*, and is marked SL. the other is for caſks *ſtanding*, and is marked SS.

Each line is to be uſed with the ſlides B and C, in the ſame manner as with the line D.

Now, the *ullage* of any caſk will be found by the following

PROPORTIONS.

1. As the *bung diameter* on C,
   Is to 100, on the proper *line* of ſeg.
   So is the *wet* or *dry* inches on C or B,
   To a *ſegment* on SL. or SS.

2. As

2. As 100 on A,
Is to the *content* of the cask on C;
So is the above found *segment* on A,
To the *ullage* on B or C.

N. B. If the quantity of liquor *remaining* in the *cask* be required, make use of the *wet* inches; if the quantity drawn *off*, the dry inches.

Examp. 1. Given a cask *lying*, bung 31 inches, content 75 gallons, wet inches $28\frac{1}{2}$; what quantity of *liqour* is in the cask, and how much drawn off.

1. For the quantity *in the cask*.

31.		100.		28.5		97.1	
C2	:	SL.	::	C2	:	SL.	

Then by the line A.

100.		75.		97.1		73.	answer
A	:	C	::	A	:	C	

2. For the quantity *drawn off*.

31.		100.		2.5		2.82	
C2	:	SL.	::	C	:	SL.	

Then by the line A.

100.		75.		2.82		2.1	answer
A	:	C	::	A	:	C	

Note. The like is to be observed by the line SS.

CHAP.

CHAP. VI.

Use of the Gauge Points *on the Line* D *of the* Artificer's *Instrument in measuring of* Polygons, *and their* Prisms.

(TAB. XIII.)

SECT. I. *Of Polygons.*

PROPORTION.

As the proper *gauge point* on D,
Is to *unity* (or 1) on C or C 2;
So is the *side* * of the given polygon on D,
To its *superficial* content on B or C.

If a circle, for side* read *diameter* or *circumference*. *See* Proportion, *chap.* IV. *sect.* 1.

N. B. To find the number of places in the answer, *see the* General Rule, *chap.* IV.

Examples.

1. By *gauge point* ⊖d: (No. 133.)

Examp. 1. Given the *diameter* of a circle $15\frac{1}{2}$ inches; what is its *superficial* content in feet?

⊖d:		1.		15.5		1.31	
D	:	C	::	D	:	C	on

Examp. 2. Given the diameter 35.8.

⊖d:		1.		35.8		6.99	
D	:	C	::	D	:	B2	on

Examp.

* *Examp.* 3. Given the diameter 7.6.

⊖d:	1.	7.6	.315	
D :	C ::	D :	C2	first *above*

* *Examp.* 4. Given the diameter 105.

⊖d:	1.	105.	60.1	
D :	C ::	D :	B	first *below*

2. By *gauge point* 8gn. (No. 142.)

* *Examp.* 1. Given the side of an *octagon*, $9\frac{1}{2}$ inches; how many *superficial feet* doth it contain?

8gn.	1.	9.5	3.026	
D :	C2 ::	D :	C2	*on*

Examp. 2. Given the side 35.

8gn.	1.	35.	41.07	
D :	C2 ::	D :	B2	first *below*

Examp. 3. Given the side 25.

8gn.	1.	25.	20.95	
D :	C2 ::	D :	C	first *below*

* *Examp.* 4. Given the side 12.

8gn.	1.	12.	4.85	
D :	C2 ::	D :	B	second *below*

SECT. II. *Of Polygonal Prisms.*

PROPORTION.

As the proper *gauge point* on D,
Is to the *length* of the prism on B or C;
So is the *side* of its base on D,
To its *content* on B or C.

See Lemma, &c. *chap.* III.

Examples.

1. By *gauge point* ⊖d: (No. 133.)

* *Examp.* 1. Given a cylinder, length 4½ feet, diameter 15 inches; how many *solid feet* doth it contain?

⊖d:	4.5	15.	5.52 answer
D :	B ::	D :	B *natural*

* *Examp.* 2. Given the length 4½ feet, diameter 3.95 inches.

⊖d:	4.5	3.95	.383
D :	B ::	D :	B2 first *above*

* *Examp.* 3. Given the diameter 9.5 inches, length 45. feet.

⊖d:	45.	9.5	22.
D :	B ::	D :	C2 second *above*

2. By *gauge point* 8gn. (No. 142.)

Examp. 1. Given an octagonal prism, length 6.4 feet, side of its base 24.6 inches; how many *solid feet* doth it contain?

8gn.	6.4	24.6	129.8
D :	B2 ::	D :	C on

* *Examp.* 2. Given the length 78. feet, side of base .875 inches.

8gn.	78.	.875	2.
D :	B2 ::	D :	C first *above*

Examp.

* *Examp.* 3. Given the length 4.35 feet, side 22.6 inches.

8gn.		4.35		22.6		74.5
D	:	B2	::	D	:	B first *below*

* *Examp.* 4. Given the length 15.4 feet, side 13.4 inches.

8gn.		15.4		13.4		92.7
D	:	C2	::	D	:	B second *below*

CHAP. VII.

Use of the Factors on B *and* C *of the* Artificer's *Instrument, in finding the* superficial *Content of* Polygons *and* Platonicks, *&c.*

TAB. XIV. and XV.

Lemma.

THE *areas* or contents of all like *superficies* are to each other, as are the *squares* of their *like* sides. Hence the

PROPORTION.

As *unity* (or 1.) on D,
Is to the proper *factor* on B or C,
So is the side of the given *polygon*, or *platonick* on D,
To the *content* on B or C.

To find the *number* of places in the *answer.*

RULE.

As many places as the *third* number in the

pro-

proportion hath $\left\{\begin{array}{l}\text{more}\\\text{less}\end{array}\right\}$ than *one*; *twice* so many places will the answer have $\left\{\begin{array}{l}\text{more}\\\text{less}\end{array}\right\}$ than the factor or *second* number, if it falls on the *collateral.* *See* General Rule, *chap.* IV.

SECT. I. *Of Factors on* B, *for* Polygons. (Tab. XIV.)

Examples.

1. By factor 12gn. (N°. 147.)

* *Examp.* 1. Given a *dodecagon* each side 15 feet; what is its superficial content in square *yards?*

1		12gn.		15.		280. answer	
D	:	B	::	D	:	B *on*	

Examp. 2. Given the side $7\frac{1}{2}$ feet.

1.		12gn.		7.5		69.9
D	:	B	::	D	:	B2 first *above*

* *Examp.* 3. Given the side .95 of a foot.

1		12gn.		.95		1.12
D	:	B	::	D	:	C2 second *above*

N. B. When the *third* number is a fraction of the *first* order, let 10 on D represent *unity*, (or 1,) then will the *answer* be *natural.* Thus in the last *example.*

1.0		12gn.		.95		1.12
D	:	C2	::	D	:	C2

2. By

2. By *factor* :Θd (N°. 157.)

Examp. 1. Given a *circle*, diameter 10.5 feet.

1.		:Θd		10.5		9.62
D	:	B	::	D	:	B *on*

Examp. 2. Given *diameter* 22.8.

1.		:Θd		22.8		45.36
D	:	B	::	D	:	C firſt *above*

Examp. 3. Given the *diameter* .75.

1.		:Θd		.75		.049
D	:	B	::	D	:	C2 ſecond *above*

Or,

1.0		:Θd		.75		.049
D	:	C2	::	D	:	C2 *natural*

SECT. II. *Of the* Factors *on* C 2 *in finding the* ſuperficial *Contents of the* Platonicks *in Square* Yards, *a Side being taken in* Feet *and decimal Parts.* (Tab. XV.)

To rectify the Inſtrument.

Place the proper *factor* on C 2 right againſt 10. on D, then ſuppoſe the ſaid *factor* to be on B, ſtanding againſt the *prime* 1. of D, and proceed as above.

Examples.

By *factor* 12rn. (No. 159.)

* *Examp.* 1. Given a *dodecaedron*, the side of one of whose pentangular planes is 14 feet; what is its superficial content in square yards?

Rectify to the proper factor. Then

1.	12rn.	14.	449.5	answer
D :	B ::	D :	B2	*on*

Examp. 2. Given the side of a *dodecadron* 3.8 feet.

1.	12rn.	3.8	33.	answer
D :	B ::	D :	B2	first *above*

N. B. When the *given side* is expressed by a *fraction* of the *first* order, let 10 on D represent *unity* or 1; then will the *answer* be *natural*.

See N. B. Example 6. *and* 7. *chap.* IV. *sect.* 1.

CHAP. VIII.

Of Proportions *of similar or* like *Areas and Superficies.*

Proportion.

As the *content* of any area or superficies on B or C,

Is to either of its given *sides* on D;

So is the *content* of any *like* area or superficies on on B or C,

To its like *side* on D.

See Lemma, *chap.* VII.

To

To find the *radius*, whereon the *third* number in the proportion is to be ſought; alſo to find the *number* of places in the anſwer.

RULE.

1. When the difference of places in the *firſt* and *third* numbers is even.

Seek the *third* number on the prime *radius*, and if it be thereon found within the compaſs of the line D, the anſwer falls on the *collateral*; if not, it falls off. Thus,

If $\left\{\begin{matrix}\text{B}\\ \text{C2}\end{matrix}\right\}$ be *prime*, and the *third* number found on its *repreſentative* $\left\{\begin{matrix}\text{C2}\\ \text{B}\end{matrix}\right\}$ the *anſwers* fall off $\left\{\begin{matrix}\textit{below}\\ \textit{above}\end{matrix}\right\}$ the *collateral*.

Note. If the middle *radius* be *prime*, the anſwer will fall *on*.

To find the *number* of places in the anſwer.

As many places as the *third* number hath $\left\{\begin{matrix}\textit{more}\\ \textit{leſs}\end{matrix}\right.$ than the *firſt*, half ſo many places will the anſwer have $\left\{\begin{matrix}\textit{more}\\ \textit{leſs}\end{matrix}\right\}$ than the *ſecond*, if it falls on the *collateral*.

If the *anſwer* falls $\left\{\begin{matrix}\textit{above}\\ \textit{below}\end{matrix}\right\}$ it, $\left\{\begin{matrix}\textit{add}\\ \textit{deduct}\end{matrix}\right\}$ *one* place.

2. When the difference of places in the *firſt* and *third* number is *odd*, then,

If the *third* number be $\left\{\begin{matrix}\textit{greater}\\ \textit{leſs}\end{matrix}\right\}$ than the *firſt*,

ſeek it on the *radius* next $\left\{\begin{matrix}\textit{above}\\ \textit{below}\end{matrix}\right\}$ the *prime*, and if it be found thereon, within the compaſs of the line D, the anſwer falls *on* the *collateral*, if not, it falls off $\left\{\begin{matrix}\textit{above}\\ \textit{below}\end{matrix}\right\}$ it.

To find the *number* of places in the anſwer,

RULE.

If the *third* number be $\left\{\begin{matrix}\textit{greater}\\ \textit{leſs}\end{matrix}\right\}$ than the *firſt*, $\left.\begin{matrix}\textit{deduct}\\ \textit{add}\end{matrix}\right\}$ *one* place $\left\{\begin{matrix}\textit{therefrom.}\\ \textit{thereto.}\end{matrix}\right\}$ Then as many places as the *third* hath $\left\{\begin{matrix}\textit{more}\\ \textit{leſs}\end{matrix}\right\}$ than the *firſt* after ſuch $\left\{\begin{matrix}\textit{deduction,}\\ \textit{addition,}\end{matrix}\right\}$ half ſo many places will the anſwer have $\left\{\begin{matrix}\textit{more,}\\ \textit{leſs,}\end{matrix}\right\}$ than the *ſecond* number, if it falls *on* the *collateral*.

If the anſwer falls $\left\{\begin{matrix}\textit{above}\\ \textit{below}\end{matrix}\right\}$ the *collateral* $\left\{\begin{matrix}\textit{add}\\ \textit{deduct}\end{matrix}\right\}$ *one* place.

Examp. 1. Given a *couch*, *floor* or *ciſtern*, length 75 inches, breadth 46, and *area* 1.6 buſhels; what muſt be the dimenſions of a like *couch*, *floor* or *ciſtern*, whoſe area ſhall be 6. buſhels?

1.6	75.	6.	145. length	
C2 :	D ::	B* :	D *above*	

And

1.6	46.	6.	89. breadth
C2 :	D ::	C2 :	D *on*

* B=C2,

Examp.

Examp. 2. Given a *piece* of *land*, length 15 chains, 60 links, breadth 5.75, *content* 9 acres; what muſt be the length and breadth of any like piece, whoſe *content* ſhall be 20 acres?

9.	15.60	20.	23.25	length
B :	D ::	C :	D	*natural*

And

9.	5.75	20.	8.57	breadth
B2 :	D ::	C2 :	D	*natural*

Examp. 3. Given an *octagon*, whoſe ſide is 9 feet and content 43.44 yards; what is the ſide of an *octagon* whoſe content ſhall be 77.2 yards?

43.44	9.	77.2	12.	anſwer
C2 :	D ::	B* :	D	*above*

* B=C2

N. B. When the *difference* of places in the *firſt* and *third* numbers do not exceed 2,

Then,

1. If $\left\{\begin{matrix}B\\C2\end{matrix}\right\}$ be *prime*, and the *third* number be $\left.\begin{matrix}\textit{greater}\\\textit{leſſer}\end{matrix}\right\}$ than the *firſt*, the anſwer will be *natural.*

2. If $\left\{\begin{matrix}B\\C2\end{matrix}\right\}$ be *prime*, and the *third* number be $\left.\begin{matrix}\textit{leſs}\\\textit{greater}\end{matrix}\right\}$ than the *firſt*; make its repreſentative *prime*, and the anſwer will be *natural.*

N. B. If the *third* number be found $\left\{\begin{matrix}\textit{above}\\\textit{below}\end{matrix}\right.$ D, ſeek it on its proper *repreſentative.*

If the middle *radius* be *prime*, and the difference of places *two*, the *third* number must be found *thereon*, and the answer will fall *on* the *collateral*, and have *one* place $\left\{\begin{matrix} \textit{more} \\ \textit{less} \end{matrix}\right\}$ than the *second*, if the *third* be $\left\{\begin{matrix} \textit{greater} \\ \textit{less} \end{matrix}\right\}$ than the *first*.

Examp. Given the *squares* $2\frac{1}{2}$ and 435. and root .845. Thus,

Place 2.5 on C2 to .845 on D; then suppose 2.5 on B to stand against .845 on —D, then it will be

2.5	.845	435.	11.14	
B :	—D ::	C2 :	+D	second *above*

Note. —D signifies the *radius* below D, and +D the *radius* above it.

SECT. III. *To find a geometrical* mean Proportional, *between any two given Numbers*, viz. *Such a Number whose* Square *shall be equal to the* Product *of the said two given Numbers.*

PROPORTION.

As *one* of the given numbers on B or C,
Is to *itself* on D;
So is the *other* on B or C,
To the *proportional* on D.

See Lemma, *part* 1. *chap.* V.

Examp.

Examp. 1. What is the geometrical *mean proportional* between 4 and 16.

4.	4.	16.	8 anſwer	
B2 :	D ::	C2 :	D *natural*	

Examp. 2. Given the *numbers* 12 and 48.

12.	12	48.	24 anſwer
C :	D ::	C :	D *natural.*

THE END OF THE THIRD PART.

A KEY

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KEY
TO THE
MODERN SLIDING RULE.

PART IV.

Description of the Line E, *with its Use in extracting the Roots of* Cubes, *and in finding the* Solidities *of the* five Platonicks *or regular Bodies; Also in finding their* Weight *in Stone, Lead, Iron, Box, and Marble; and in* Proportions *of similar or like* Solids.

CHAP. I.

Description of the Line E, *and of its Use in extracting the Roots of* Cubes.

SECT. I. *Description.*

THIS line is put on the *back* sides of the slides B and C, in a broken and doubled manner. It consists of four complete equal and alike *radii* of numbers, called E, E 2, E 3, and E 4. with part of another called E 5, and distinguished by the difference of the number of cyphers annexed to the *prime* 1. of each *radius*; any

three of which are exactly equal in length to the *radius* D. Thus,

Place the slides between the parts of the *radius* D, and move them together till the *prime* 1. of the *radius* E, stands right against the *prime* 1. of the *radius* D. Then will the *prime* 1. of the *radius* E 4. viz. the point marked 1000. on the *lower* edge of the slide, stand *right* against the point 10. on the *lower* edge of D.

This I call the *direct* position of the *slide* or *radius* E.

Now, from what hath been said on the line D, (*chap.* I.) it is easy to conceive that the *upper* edge of the slide E, and the *lower* edge of the slide E 2, do represent each other; also, that the *upper* edge of the slide E 2, and *lower* edge of the slide E, are representatives of each other.

SECT. II. *Of the Disposition of the* Primes *and* Intermediates *on the Line* E.

These are disposed in such manner, that when the slide E is in *direct* position, it becomes with the line D, a table of *cubes* with their roots.

Place the slide E direct, now have you all cubes represented on the slide E, and their roots on D.

Thus,

Against 8 on *radius* E is its *root* 2. on D; against 27. on E 2, is its root 3 on D; against 64. E 2, is 4. its root, and against 729. on E 3, is its *root* 9. on D, &c. Hence,

SECT.

SECT. III. *To find the* Radius *whereon to ſeek any given* Cube *or* Number.

1. For *integer* or *mixed* dumbers.

RULE.

If the given *cube* conſiſts of $\left\{\begin{matrix}1,\\2,\\3,\end{matrix}\right\}$ *integral* places, it muſt be ſought on the *radius* $\left\{\begin{matrix}\text{E.}\\\text{E2.}\\\text{E3.}\end{matrix}\right.$

N. B. If the given cube conſiſteth of *more* than 3 *integral* places, divide the number of places by 3. and if nothing remains, the cube muſt be ſought on E 3.

If the remainder be $\left\{\begin{matrix}1.\\2.\end{matrix}\right\}$ ſeek it on radius $\left\{\begin{matrix}\text{E.}\\\text{E2.}\end{matrix}\right.$

2. For *fractional* cubes.

1. If the *cube* be of the $\left\{\begin{matrix}\text{firſt}\\\text{ſecond}\\\text{third}\end{matrix}\right\}$ order, ſeek it on radius $\left\{\begin{matrix}\text{E3.}\\\text{E2.}\\\text{E.}\end{matrix}\right.$

2. If it be of any other order, divide the number of cyphers prefixed by 3; if nothing remains, the *cube* muſt be found on the radius E 3.

If $\left\{\begin{matrix}1\\2\end{matrix}\right\}$ remains, ſeek it on $\left\{\begin{matrix}\text{E2.}\\\text{E.}\end{matrix}\right.$

SECT. IV. *To find the* Number *of Places in the* Root *of any given Cube.*

1. For *integral* and *mixed* cubes.

It is manifeſt from what hath been ſaid, that if the given number conſiſteth of three *integral* places, its root will conſiſt of one *integral* place, viz. one *third* of the number of places in the given *cube*.

Now, ſeeing the radius D is equal in length to three radii of E, therefore, for every *three* places any given *cube* is ſuppoſed to be increaſed or decreaſed, the *root* of ſuch *cube* muſt be ſuppoſed to to be encreaſed or decreaſed reſpectively by *one* place, by Corollary, *part* 1. *chap.* III. Hence the

RULE.

If the given *cube* conſiſteth of one, two or three *integral* places, its root will conſiſt of *one* integral place.

If the *cube* conſiſteth of *more* than *three* integral places, divide the ſaid number of places by 3, and if there be no remainder, the quotient figure will ſhew the number of places in the *root*, but if any thing remains, the *root* will have *one* place *more*.

2. For *fractional* cubes.

It appears by this foregoing ſection, that if the *cube* given be of the firſt, ſecond, or third order, its *root* will be of the *firſt* order.

N. B. If the *cube* given be of any other order, divide

divide the number of *cyphers* prefixed by *three*, and if the quotient figure be $\left\{\begin{array}{l}1.\\2.\\3.\end{array}\right\}$ the *root* will be a fraction of the $\left\{\begin{array}{l}\text{2d}\\\text{3d}\\\text{4th}\end{array}\right\}$ order.

CHAP. II.

Use of the Factors *on the Line* E, *in finding the* Solidities *of the five* Platonicks *or regular Bodies*; *and also their* Weight *in Stone, Box, and Marble, &c.*

N. B. EACH *factor* on the radius E, is no other than the solidity of its corresponding *platonick*, whose side is *unity*, or 1. and each *factor* on the radius $\left\{\begin{array}{l}E2,\\E3,\\E4,\end{array}\right\}$ is the weight of its corresponding *platonick* of $\left\{\begin{array}{l}\text{common stone}\\\text{box}\\\text{marble}\end{array}\right\}$ in *pounds averdupoise*, whose side is *one inch*.

Lemma.

The *solidity*, and also the *weight* of all similar or like solids are in proportion to each other, as are the *cubes* of their *homologous* or *like* sides. Hence the following

GENERAL PROPORTION.

As unity on D
 Is to the proper *factor* on the line E.
So is the side* of the given *platonick* on D,
 To the answer on E.

* *N. B.* By the side of any *platonick* is meant the side of one of the *planes*, whereof its *superficies* is composed.

To *find* the *number* of *places* in the *answer*.

Note. In all oblique positions of the slides, there will be *two* entire *radii*, and also *parts* of *two* others of the line E, within the compass of the line D, viz. the *two* entire *middle radii*, and parts of the *two extremes*, consequently the answer hath *four* varieties, viz.

It may fall on { on / on the next / on the second / on the third } { radius above } { the collateral. }

Hence the

RULE.

1. If the *third* number in the proportion consisteth of *equal* places with the *first*, viz. one, the answer will consist of *equal* places with the *second* or given *factor*, if it falls on the *collateral*.

2. If the *third* number consisteth of *unequal* places with the first, then, because the *radius* D is equal in length to *three* radii of E, it will be,

As

As many places as the *third* hath $\left\{\begin{matrix}\textit{more}\\ \textit{less}\end{matrix}\right\}$ than the *first*, *three times* so many places will the *answer* have $\left\{\begin{matrix}\textit{more}\\ \textit{less}\end{matrix}\right\}$ than the *second*, if it falls on the *collateral*.

If the answer falls on the $\left\{\begin{matrix}\text{first}\\ \text{second}\\ \text{third}\end{matrix}\right\}$ radius above, add $\left\{\begin{matrix}1\\ 2\\ 3\end{matrix}\right\}$ places.

N. B. The *difference* of the radii is easily distinguished by the *difference* of the number of *cyphers annexed* to the prime 1. of each radius.

Examples.

1. *By the factors on radius* E, *for solidities.*

(Tab. XVI.)

1. 1. By *factor* .4rn (No. 166.)

Examp. 1. Given a tetraedron, whose side is 6; what is its solidity?

1.	.4rn	6.	25.4	answer
D :	E ::	D :	E3	second *above*

Examp. 2. Given the side of a tetraedron 12.

1.	.4rn	12.	203.5	
D :	E ::	D :	E	*on*

The *third* number hath *one* place more than the *first*; ∴ the *answer* hath *three* more than the *second*.

Or thus, natural,

1	.4rn	12.	203.5
D :	E ::	+D :	E4

See *Examples* 6 and 7, *part* III. *chap.* IV.

2. *By factors for the* weight *of platonicks.*

Of *rectifying* the instrument.

Place the slide E *direct*; (*See sect.* 1.) now against the prime 1. of the radius E 2, is a brass pin on D, marked G, whose distance from the prime 1. of the radius D, is exactly equal to the length of one radius of the line E; hence, in all *oblique* positions of the slides, what ever *prime* or *intermediate* of E 2 stands against the said point G, the like *prime* or *intermediate* of the *radius* E, will stand against the prime 1. of the *radius* D; and it is equally obvious, that whatever point of E 3 stands against the point G of *lower* D, the like point of E will stand against the prime 1. and also the point 10. on D. That is,

In all *oblique* positions of the *slides*, the prime 1. the 2 points G, and the point 10. on D, do stand against *like* primes or intermediates. Hence,

To rectify the instrument.

RULE.

1. For *factors* on E 2 or E 3.

Place the proper *factor* against the point G, and then suppose it to stand against the prime 1. of D, and proceed as above.

2. For *factors* on E 4.

Place the proper *factor* against the point 10. on D, then suppose it to be on E.

Examp.

Examples.

1. By *factors* on E 2, for *stone*. (Tab. XVII.)

1. By *factor* :Spd (N°. 176.)

Given a *sphere*, whose diameter is 7.5 inches; what is its weight in *pounds* averdupoise?

Rectify as above taught. Then

1.		:Spd		7.5		20. answer	
D	:	E	::	D	:	E4	third *above*

2. By *factor* .12rn (N°. 177.)

Given a *dodecaedron* whose side is 10.5.

Rectify. Then

1.		.12rn		10.5		801. answer	
D	:	E	::	D	:	E	*on*

The *third* number hath one place *more* than the *first*; therefore the *answer* hath three *more* the *second*.

Or thus, natural,

1		.12rn		10.5		801.
D	:	E	::	+D	:	E4

See *Example* 6 and 7, *part* III. *chap*. IV.

By *factors* on E 3, for *box*. (Tab. XVIII.)

By *factor* :8rn (N°. 179.)

Given an *octaedron*, whose side is 34.5 inches.

Rectify. Then

1.		:8rn		34.5		72.1	
D	:	E	::	D	:	E2	first *above*

By *factors* on E.4 for marble. (Tab. XIX.)

By *factor* .12rn (No. 191.)

Given a *dodecaedron*, whose side is 8.45.

Rectify.

1.		.12rn		8.45		4.3.	
D	:	E	::	D	:	E4	third *above*

CHAP. III.

Of Proportion *of* Solids *by the Line* E.

GENERAL PROPORTION.

AS the *content* or *weight* of any given *solid* on E,
Is to its *side* on D;
So is the *content* or *weight* of any *like* solid on E,
To its like *side* on D.
See Lemma, *chap.* II.

N. B. The *radius* whereon to find the *third* number in the proportion, and also to know the *number of places* in the answer, may be known from what hath been said in the foregoing chapter, and *N. B. chap.* VIII. of the line D.

Examp. 1. Given a *parallelopepid* length 124 inches, breadth 18, depth 11, content or solidity 87, ale gallons; what must be the dimensions of a like

a like parallelopepid, whose content shall be 100. gallons ?

	87.		124.		100.		129.8=length	
I.	E	:	D	::	E2	:	D	
	87.		18.		100.		18.85=breadth	
II.	E2	:	D	::	E3	:	D	*natural*
	87.		11.		100.		11.52=depth	
III.	E	:	D	::	E2	:	D	

Examp. 2. Given a *parallelopiped*, length 5. feet, breadth $4\frac{1}{2}$, depth 2, content 45 feet; what must the dimensions of a like *parallelopiped* be, whose content shall be 150. feet?

	45.		5.		150.		7.47=length
I.	E3	:	D	::	E4	:	D
	45.		4.5		150.		6.72=breadth
II.	E3	:	D	::	E4	:	D
	45.		2.		150.		2.98=depth
III.	E2	:	D	:	E3	:	D

Examp. 3. Given a *ship of war*, length of keel 80 feet, breadth of midship-beam 30, depth 15, *burthen* 360 tons; what must be the *dimensions* of a like built ship, whose burthen shall be 1000 tons ?

	360.		80.		1000.		112.4=length	
I.	E4	:	D	::	E5	:	+D	*above*
	360.		30.		1000.		42.17=breadth	
II.	E3	:	D	::	E4	:	D	*on*
	360.		15.		1000.		21.08=depth	
III.	E2	:	D	::	E3	:	D	*natural*

Examp. 4. Given an *iron bullet*, diameter 4 inches, weight $9\frac{1}{4}$ *pounds*; what must the *diameter* of a bullet of the same metal be, whose weight shall be 30 *pounds*?

$$\begin{array}{ccccccc} 9.25 & & 4. & & 30. & & 5.925 \text{ answer} \\ E2 & : & D & :: & E3 & : & D \end{array}$$

THE END OF THE FOURTH PART.

A KEY

A

KEY

TO THE

MODERN SLIDING RULE.

PART V.

Containig the Construction *and Use of Tables of* natural *Sines and Tangents: Also, the Manner of working* Proportion *by the* sliding *Sines and Tangents, and of their Use in* plane *Trigonometry.*

CHAP. I.

Of the Construction *of Tables of* natural *Sines and Tangents.*

THESE are tables shewing what *proportion* the sine or tangent, &c. of any given arch of a circle, bears to the *radius* of the said circle, which are thus constructed.

1. Natural *sines.*

1. On the centre C, with any *radius* (AC) describe the quadrantal *arch* of a circle AD; and compleat the quadrant ACd.

2. *Divide*

2. *Divide* the arch AD into 6 equal parts in the the *points* b, c, d, e, and f, so will the said *points* be just 15 *degrees* distant from each other, and the

right sine	gb	will be the sine of	15.	degrees, viz. of the arch	Ab	or angle	ACb	whose co-sine is	bb.
	hc		30.		Ac		ACc		cc.
	id		45.		Ad		ACd		dd.
	ke		60.		Ae		ACe		ee.
	lf		75.		Af		ACf		ff.

3. Divide the *radius* CD into 10. equal parts, and number the divisions 1, 2, 3, 4, 5, &c. from C to D.

Now, the sines gb, hc, id, &c. being applied to the *radius* CD, will point out thereon, what proportion each *sine* doth bear to the *radius* CD, which proportion is the *natural* sine of its respective arch or angle. Thus,

The sine	gb	will reach from C to	2.588190	on radius CD, the *natural* sine of	15.	degrs.
	hc		5.000000		30.	
	id		7.071068		45.	
	ke		8.660254		60.	
	lf		9.659258		75.	

Now it is very conceiveable, that, if the *radius* CD, be supposed to be divided into 100, 1000, or 10,000 equal parts, the above *sines* will bear the same *proportion* thereto, as they now do to the radius 10. or CD. That is,

If the radius CD be {100, 1000,} then will the *sine* gb be {25.88190, 258.8190} and so on.

Consequently,

Consequently,

If the radius be 10000, the *natural* sine of	15.	degrees will be	2588.190
	30.		5000.000
	45		7071.068
	60.		8660.254
	75.		9659.258

Hence, if the *radius* CD be supposed to be divided into 10000. equal *parts*, and the arch AD into 90. equal *parts* or degrees, and each *degree* into 60. equal parts or *minutes* of a degree; and the *cosine* of each minute be drawn to the *radius* CD, the said *cosines* will point out on CD, what proportion the *sine* of each minute doth bear to the said *radius* CD, and will be in effect the same table of of *natural* sines, which you may find in those excellent *Mathematical Tables* constructed by Messrs. *Briggs*, *Wallis*, *Halley*, and *Sharp*, late Savillian professors of *geometry* at Oxford, and published by the ingenious Mr. *Sherwin*.

2. Of natural *tangents*.

1. On the centre C, with any *radius*, CB, draw the arch BD, and compleat the *quadrant* BCD.

2. Divide the *arch* BD into 6. equal parts, as the former, in the points a, b, c, d, e.

3. Draw the *tangent* line BE, and from the *center* C, through the points a, b, c, d, e, draw the *secants* Cg, Ch, Ci, Ck, and CE;

so will	Bg	be the *tangent* of	15	degrees, viz. of the *arch*	Ba	or angle	BCa.
	Bh		30		Bb		BCb.
	Bi		45		Bc		BCc.
	Bk		60		Bd		BCd.
	BE		75		Be		BCe.

Now,

Now if the *tangent* $\left\{\begin{matrix}\text{Bg}\\ \text{Bh}\\ \text{Bi}\end{matrix}\right\}$ be applied to the *radius* CD, it will reach from C to $\left\{\begin{matrix}2.679492\\ 5.773503\\ 10.000000\end{matrix}\right\}$ the natural *tangent* of $\left\{\begin{matrix}15\\ 30\\ 45\end{matrix}\right\}$ degrees.

Again, continue the *radius* CD, and lay off DG, GH, HF, each equal to CD, and suppose the whole line CF to be divided into 40 such *equal* parts as of which the *radius* CD is 10. Then

The *tangent* $\left\{\begin{matrix}\text{Bk}\\ \text{BE}\end{matrix}\right\}$ being applied to CF, will reach from C to $\left\{\begin{matrix}17.320508\\ 37.320508\end{matrix}\right\}$ the natural *tangent* of $\left\{\begin{matrix}60\\ 75\end{matrix}\right\}$ degrees.

Hence, if the *radius* CD be supposed to be 10000, the line CF will be 40000; and

Consequently,

If the *radius* be 10000, the *natural* tangent of $\left\{\begin{matrix}15\\ 30\\ 45\\ 60\\ 75\end{matrix}\right\}$ degrees will be $\left\{\begin{matrix}2679.492.\\ 5773.503.\\ 10000.000.\\ 17320.508.\\ 37320.508.\end{matrix}\right.$

Hence, if the *radius* CD, be supposed to be divided into 10000 equal parts, and the line CF to be *infinitely* continued, divided and numbered respectively, and the *tangent* line BE, be also supposed to be *infinite*, and graduated to every minute of a degree; then, if from each minute of the line BE, a right line be drawn *perpendicularly* on the

the *infinite* line CF, it will point out thereon what *proportion* the tangent of each minute of a degree, doth bear to the *radius* CD, and will be in effect, Mr. *Sherwin's* Table of *Natural tangents*.

N. B. Tables of natural *secants* may be formed alſo by transferring the *secant* of each minute of the quadrant BCD, to the *infinite* line CF, and comparing them therewith. Thus,

$$\left.\begin{array}{l}Cr\\Cf\\Ct\\Cu\end{array}\right\} = \left\{\begin{array}{l}11.547\\14.142\\20.000\\38.637\end{array}\right\} \text{is the natural } \textit{secant} \text{ of } \left\{\begin{array}{l}30.\\45.\\60.\\75.\end{array}\right\} \text{degrees, the } \textit{radius} \text{ CD being 10.}$$

N. B. The *verſed ſines* Ag, Ah, Ai, Ak, &c. may in the like manner be transferred to the line CF.

SECT. II. *Of the Uſe of Tables of* natural *Sines and Tangent in working of* Proportions.

1. Of *tangents*.

If in any *triangle*, a right line be drawn *parallel* to either of its ſides, ſo as to cut the other two ſides, ſuch *parallel* will cut off a *triangle* like unto the *firſt*, whoſe ſides will bear the ſame *proportion* to each other reſpectively, as do the ſides of the *firſt* *; and whoſe *angles* will be equal to the *angles* of the *firſt* †.

Thus, from point F in *radius* CB, draw the right line or *ſine* Fe, *parallel* to the tangent line

* Second Propoſition of Euclid, book 6.
† Fifth Propoſition of the ſame book.

BE; then will the *triangle* CeF, be like to the *triangle* CEB; and the *sides* of the triangle CeF, will bear the same *proportion* to each other, as the *sides* of the triangle CEB, do to each other: that is,

As CB is to BE, so is CF to Fe; and as CB is to CE, so is CF to Ce.

Or thus alternate:

As CB is to CF, so is BE to Fe; and as CB is to CF, so is CE to Ce.

Or thus:

On the center C, draw the *arch* Fg; then will Fe, be the *tangent* of the angle FCe, to the *radius* CF; and consequently doth bear the same *proportion* to CF, as the *tangent* BE doth to CB; that is, as the *radius* CB is to the *radius* CF, so is the *tangent* BE, to the *tangent* Fe.

Now the *radius* CF is the *sine* of 15 degrees, viz. the *cosine* of ∠ eCF, viz. 75 degr. = 2.588190.

Therefore it will be,

As the *radius* CB = 10, is to *radius* CF = 2.588190; so is the *tangent* BE = 37.320508 to a *fourth* geometrical proportional, which will be the tangent Fe.

Thus,

rad. *tang.*
10. : 2.588190 :: 37.320508 : 9.659258. = Fe
×2.588190

10)96.59258(9.659258

which by the table of *natural* tangents will run thus,

rad. *tang.*
10000. : 2.588190 :: 37320.508 : 9.659258

that is, if the line Fe be applied to the line CF, it

it will reach from C to the point 9.659258 thereon. Hence,

Having either *leg* of a *right* angled triangle, and its adjacent *oblique* angle, the other will be found by the following

PROPORTION.

As the *radius* 10000,
Is to the given *leg*,
So is the *tangent* of the adjacent *angle*,
To the other *leg*.

That is, *multiply* the given *leg* by the *natural* tangent of its *adjacent* angle, and *divide* the product by the natural *radius* 10000, the *quotient* will be the other *leg*.

Examp. 1. Given one *leg* of a *right* angled triangle 3. and its *adjacent* angle 53. degrees, 8. minutes, to find the other *leg*.

First seek the *natural* tangent of 53° 08′ in the tables, which will be found to be 13334.900.

Therefore it will be

rad. *tang.*
10000 : 3. :: 13334.900 : 4. = answer
×3

10000) 40004.700 (4.0004

Examp.

Examp. 2. Let the given *leg* of a *right* angled triangle be 4, and its *adjacent* angle 36. degrees, 52. minutes; to find the other *leg*.

The *natural* tangent of 36° 52′=7500.000.

Therefore

rad.		*tang.*		
10000. :	4 ::	7500.000 :	3.=answer	
	4			

10000)30000.000(3.000

Note. If the *hypothenuse* be required, then the table of natural *secants*, must be used instead of that of *tangents*.

Examp. Given the *leg* and angle, as in the last example, to find the *hypothenuse*.

First seek the natural *secant* of 36° 52′ which will be found to be 12499 471.

Therefore,

rad.		*sec.*	
10000. :	4. ::	12499.471 :	5.=answer.

2. Of natural *sines*.

If the *hypothenuse* of any *right* angled triangle, be made the *radius* of a circle, then will the sides of that triangle be to each other, as are the *sines* of their *opposite* angles. Thus,

If in the triangle CBE, the *hypothenuse* CE be made the *radius*, then I say, the sides CE, EB, and BC, will be to each other as are the *sines* of their *opposites* angles.

DEMON-

DEMONSTRATION.

It hath been above proved that the *ſides* of the triangle CEB, are to each other reſpectively as the *ſides* of the triangle CeF are to each other.

But the *ſides* of the triangle CeF, are the *ſines* of their reſpective *oppoſite* angles. Thus,

Draw eg *parallel* to CF; ſo will eg be the ſine of the angle DCe, and Fe the *ſine* of the angle FCe. (*See ſect.* 1.) Now, *angle* CeF=*angle* ebg, (29 E 1) and FC=eg (33 E 1) and Ce=rad. CD (Def 15 E 1.) Therefore CF is the *ſine* of its *oppoſite* angle CeF. Hence, the *ſides* of every *plane* triangle are to each other, as are the *ſines* of their *oppoſite* angles.

Now, if the *hypothenuſe* of any triangle be ſuppoſed to be *radius*, and divide into 10000 equal parts; then will either *leg* of the ſaid triangle be found in the table of *natural* ſines, againſt the degree and minute of that *angle* whereof it is the *ſine*.

So in the triangle CEB, if CE be 10000, the *leg* EB will be expreſſed in the table by the *natural* ſine of 75 degrees its oppoſite angle; and the ſegment CB, by the *natural* ſine of 15 degrees its oppoſite angle.

Hence,

Firſt, if the *hypothenuſe* and one of the *oblique* angles of any *right*-angled triangle be given, the *ſide* oppoſite to the given angle will be found by the *natural* ſines, by the following

PROPORTION.

As 10000, the *natural* radius or sine of 90,
Is to the *hypothenuse*;
So is the *natural sine* of the given angle,
To its *opposite* side.

Thus in the *triangle* CeF, let the *hypothenuse* Ce be 10, and the given *angle* be FCe=75 degrees, to find the *side* Fe.

Thus,

rad. *sin.* 75.
10000 : 10 :: 9659.258 : 9.659258=Fe

Secondly, if the *hypothenuse* and one of the *legs* be given, the angle *opposed* to the given *leg* will be found by this

PROPORTION.

As the given *hypothenuse*,
Is to the *radius* 10000;
So is the given *leg*,
To the sine of its *opposite* angle.

Thus in the aforesaid triangle, given the *hypothenuse* Ce 10, and the *leg* Fe 9.659258, to find the *angle* C.

Thus,

rad. *sine.*
10 : 10000 :: 9.659258 : 9659.258=angle 75°

N. B. If the given triangle be *oblique* angled, the *radius* will not be concerned; but the *sides* and *angles* of such triangle will be found by the following

PRO-

PROPORTIONS.

II. As the given *side*
Is to the *natural* sine of its *opposite* angle,
So is either of its other *sides*
To the *natural* sine of its *opposite* angle.

And,

II. As the *natural* sine of the given angle,
Is to its *opposite* side;
So is the *natural* sine of either of the other angles,
To its *opposite* side.

CHAP. II.

Description *of the Instrument of* sliding *Sines and Tangents, with the* Estimation *of Primes and Intermediates thereon, and of the* Radius.

SECT. I. *Description.*

THIS instrument is in the usual form of a *parallelopepid* of four planes or sides, of about 15 inches long.

On one of its broader planes is put a line of *sines*, marked *Sin.* or S. This line I shall distinguish by the line G.

On the opposite *plane* or side to this, is put a line of *tangents*, marked *Tan.* or T. This I call the line K.

On one of the narrower *planes*, viz. that next under the plane *sines* are put *two radii*, and

part of a *third* radius of the line of numbers, marked *Num.* or N. and called line A.

On the oppoſite *plane* to this, is put a line of *verſed* ſines, marked *V. Sin.* or V. S.

N. B. All theſe lines are put on in a *broken* and *doubled* manner, as the line D on the other inſtruments, viz. part on the *upper* and part on the *lower* edges of the ſeveral planes.

SECT. II. *Of the Slides.*

To this inſtrument belong four *ſlides* or ſliding *rods*, each equal in *length* to the inſtrument.

On one ſide of *two* of theſe ſlides, called and marked B and C, are put *three radii* of numbers, and part of a *fourth*. The *radii* on C are exactly alike to thoſe on *plane* A; and thoſe on B are the ſame continued.

Theſe are to be uſed together, with planes, *ſines*, *tangents*, and *numbers*, in *plane* trigonometry.

N. B. The difference of the *value* of the primes and intermediates on each of theſe *radii*, and alſo of thoſe on plane A, is diſtinguiſhed by the *difference* of the number of *cyphers* annexed to the prime 1. of each *radius*.

On one ſide of the other two *ſlides* is put a line of *ſines* marked *Sin.* or S. The left-hand ſlide, having on it the *leſſer* ſines, I call H; and the other I. The *line* or *ſlide* I is exactly alike unto *plane* G, and on *ſlide* H are the ſame lines continued.

Both

Both theſe ſlides are to be uſed together, with planes, *ſines*, and *tangents* in *ſpheric*, and with *ſlide* A in *plane* trigonometry.

On the backſide of theſe is put a line of *tangents*, marked *Tan.* or T. The *ſlide* at the left-hand, is called L. and the other M. The *ſlide* M, is exactly alike unto *plane* K, and the *ſlide* L, is the ſame line continued.

Both theſe *ſlides* are to be uſed together, with the planes, *ſines* and *tangents* in *ſpheric*, and with ſide A, in *plane* trigonometry.

SECT. II. *Of the* Eſtimation *of Primes and Intermediates on the Lines of* Sines *and* Tangents.

1. The *primes* 10. 20. 30. &c. on the *upper* edge of planes, *ſines* and *tangents*, and alſo on the *upper* edges of the *ſlides* I and M, do repreſent *tens* of minutes of a degree.

2. The *primes* 1. 2. 3. on *upper*, and 4. 5. 6. &c. up to 9. on *lower* edge of both *planes*, and alſo of *ſlides* I and M repreſent *units* of degrees; and the primes 10, 20, 30, &c. on the *lower* edges repreſent *tens* of degrees.

N. B. The *ſlide* $\left\{\begin{matrix}\text{H.}\\ \text{L.}\end{matrix}\right\}$ is only the *ſlide* $\left\{\begin{matrix}\text{I.}\\ \text{M.}\end{matrix}\right.$ continued.

2. *Intermediates* are of two ſorts, viz. *greater* and *leſſer*, and are to be eſtimated according to what number of *each* there are between any two *primes*, and the *difference* of the value of thoſe *primes*.

 Thus

Thus on the lines of *fines*;

From 30 minutes, to 1 degree, each *greater* intermediate doth represent 1 minute.

Each *greater* intermediate between	degrees and		doth represent	and each *lesser*
1.		10.	10 min.	5. min.
10.		20.	1 degr.	10. 15. 30.
20.		30.		
30.		60.		
60.		80		
80.		90.	30 min.	

And thus on the line of *tangents*.

The *intermediates* up to 30. degrees, are the fame, as are the *fines*; but from 30. degrees, to 45. each $\left\{\begin{matrix}\textit{greater}\\ \textit{lesser}\end{matrix}\right\}$ intermediate reprefents $\left\{\begin{matrix}1\ \textit{degr.}\\ 15\ \textit{min.}\end{matrix}\right.$

N. B. The *intermediates* on the *upper* edges of flides I and M, do each reprefent 1. *minute*.

N. A. The *intermediates* on the *lower* edge of $\left\{\begin{matrix}H\\ L\end{matrix}\right\}$ are exactly the fame with thofe on the *upper* edge of $\left\{\begin{matrix}I.\\ M.\end{matrix}\right.$

Note. Every *fubdivifion* is not put on the annexed *plate.*

SECT. III. *Of the* Radius.

By the *radius* here, is not to be underftood any *part* or *portion* of either of the lines of *fines* or *tangents*; but a certain *point* in each: and becaufe the *fine* of 90. and the *tangent* of 45 degrees of every circle, is equal to the *radius* or femidiameter of the faid *circle*, therefore each of thefe *points* on the inftrument, is called the *radius*, and doth always reprefent the *radius* of fome *circle*.

CHAP.

CHAP. III.

Of the Disposition *of the Primes and Intermediates on the Lines of* Sines G, H, *and* I, *and how to find the* natural *Sine of any arch thereby.*

SECT. I. *Of the plane* Sines G.

THESE are put on in such manner, as that with the slides B and C, they compose a *table* of natural sines. Thus:

Place B, C, proper between the two parts of the line of *sines* G, move them together till 1000. on *slide* C, stands right against the *radius* or sine of 90 degrees on the *lower* edge of G.

Now, you are to suppose the sines 3, 2, 1, *degrees*, 50, 40, 30, &c. *minutes* on the *upper* edge of G, to be continued down from the *sine* 4 degrees on its *lower* edge, and standing against the *lower* edge of B, in the same manner as the said sines on *upper* G, doth against C.

Hence the slides B and C, when used with the plane *sines*, do represent 3 distinct *radii* of numbers, with part of a *fourth*. Hence also the *lower* edge of B, and upper edge of C, do represent each other. (*See* Line D.)

And in this *position* of the slides you have a *table* of natural *sines* to the radius 1000.

Thus if the *radius* of any circle be 1000, the

 natural

	D.	M.			
natural sine of	74.	22	will be	963.	on C, and so on as appears by the instrument.
	12.	07		209.9	
	5.	43		99.6	
	1.	49		31.71	
	0.	30		8.726	
	0.	10		2.908	

Now, if you suppose 1000. on C, to be 10000. then will the *instrument* become Mr. *Sherwin's* Tables of *natural* Sines, down to the sine of 10 minutes.

SECT. II. *Of the Slides Sines* H *and* I.

These taken together, are a line of *sines* exactly alike unto G, and continued down on the *upper* edge of H, to 1 minute of a *degree*.

Thus, place the slides H and I *proper* between the two parts of the line G, and move them together, till the *sine* 90. on I, stands against *sine* 90 on G; then will every degree and minute of G stand against its like degree and minute of the slide I.

Now, in the same *position* of the slides you are to suppose the *primes* 3. 2. 1. degrees, 50. 40. 30. minutes, &c. on *upper* edge of G, to be continued to the *left* hand from prime 4, on its *lower* edge, and standing against their *like*, on the *lower* edge of H; hence the *upper* edge of I and the *lower* edge of H, do represent each other.

Again, in this *position* of the slides, you are to suppose the *upper* edge G, to be continued down

to

to the *left* hand, having thereon the primes 3. 2. 1. with their intermediates, ſtanding againſt their like, on *upper* edge of H.

SECT. III. *How to find the* natural *Sine of an Arch or Angle* leſs *than* 10 *minutes.*

1. Place 1000 on C to the *radius* or ſine 90 on G; then againſt 10 minutes on G, is 2.908 its *natural ſine* on B.

2. Place 10 minutes on *ſlide* H to 29.08 on plane A, and ſuppoſe 29.08 to be 2.908, viz. the *natural ſine* of 10 minutes, then againſt any minute on H, is its *natural* ſine on A.

Thus againſt	8.	minutes on H, is its natural ſine	2.327	on A.
	6.		1.745	
	4.		1.163	
	1.		.2909	

Note. The like is to be obſerved of the *lower* natural *tangents.*

CHAP. IV.

Of Proportions by the Lines *of* Sines *and* Numbers.

THESE may be divided into two ſorts, viz. of *ſines* to numbers, and of *numbers* to ſines.

SECT. I. *Proportions of* Sines *to Numbers.*

How to find a fourth geometrical proportional *number*, to two given *ſines* and a given *number*.

PRO-

PROPORTION.

*As the first given *sine* on G,
Is to the given *number* on B or C;
So is the other given *sine* on G,
To the fourth *proportional* on B or C.

Examp. 1. Given the *sines* of 40 minutes and 3 degrees, and the *number* 35, to find the other number or *fourth proportional*.

Thus:

40′		35.		3°		157.4 answer
G	:	C	::	G	:	C first *above*

See N. B. chap. I. *sect.* 2.

Examp. 2. Given the *sines* 40 minutes and 9 degrees, 45 minutes, and *number* 35, to find the other *proportional* number.

40′		35.		9°.45′		509.4 answer
G	:	C	::	G	:	C first *above*

Examp. 3. Given the *sines* 40 minutes and 45°30′, and number 3.5; to find the other number.

40′		3.5		45°30′		214.5=answer
G	:	C	::	G	:	C second *above*

Examp. 4. Given the *radius* or *sine* of 90 degrees, and the sine of 10 minutes, and *number* 365; to find the fourth proportional.

90°		365.		10′		1.061=answer
G	:	C	::	G	:	B third *below*

SECT.

SECT. II. *Proportion of* Numbers *to find*

How to find a fourth *proportional* sine to two given numbers and a given sine.

This is only the *converse* of the former; hence the following

PROPORTION.

As the first given *number* on B or C,
Is to the given *sine* on G;
So is the other given *number* on B or C,
To the fourth *proportional* on G.

Examp. 1. Given the numbers 25. and 8.7, and the sine of 50 degrees, 30 minutes, to find the other proportional sine.

25.		50°30′		8.7		15°34′=answer
C	:	G	::	C	:	G

Examp. 2. Given the numbers 25. and 1.5 and sine 50°30′; what is the other proportional sine?

25.		50°30′		1.5		2°39′=answer
C	:	G	::	C	:	G

Examp. 3. Let the given *numbers* be 250. and .55, and the *sine* 60°30′, to find the other *proportional.*

This requires two operation. Thus,

	250.		60°30′		.835		10′
I.	C	:	G	::	B	:	G
	.835		10′		.55		96′
II.	A	:	H	::	A	:	H

See sect. 3. *chap.* III.

CHAP.

CHAP. V.

Solution of Problems in plane *Trigonometry by the Lines of* Sines.

FROM what hath been said, it appears, that if the *primes* and *intermediates* of the slides B and C, be supposed to represent the *sides*, and those of the line G the *angles*, of any right lined triangle. Then,

All problems relating to *plane* trigonometry, wherein *sines* only are concerned, will be solved on the lines G, B and C, by the following

PROPORTIONS.

I. As the *sine* of any given angle on G,
Is to its opposite *side* on B or C;
So is the *sine* of either of the other two angles on G,
To its opposite *side* on B or C.

And,

II. As either of the *sides* of any given triangle on B or C,
Is to the *sines* of its opposite angle on G;
So is either of its other *sides* on B or C,
To the *sine* of its opposite angle on G.

Examples.

1. To find a side.

Examp. 1. In the *oblique* triangle ABC, (plate 1. fig. 2.) given the *angle* A 41°49′, the *angle* B

65°14′;

65°14′; and the *side* AC 429, opposed to one of them; to find the *side* opposed to the other.

Thus,

Sin. 65°14′	429,	Sin. 41°49′	315.=side BC
G :	C ::	G :	C *on* collat.

Examp. 2. Given in the same triangle, the *angles* A 41°49′, and C 72°53′, and the *side* CB 315, to find the *side* AB.

Sin. 41°49′	315.	Sin. 72°53′	451.5=answer
G :	C ::	G :	C

Examp. 3. In the *rectangled* triangle BCD, (plate 1. fig. 3.) there are given the *angles* B 36°52′ and D 53°08′, and the side BC 476; to find the *side* CD.

Sin. 53°08′	476.	Sin. 36°52′	357.=answ.
G :	C ::	G :	C

Examp. 4. Given the *right* angle C, the angle B 36°52′, and the *hypothenuse* Bd 595, to find the *side* Cd.

Rad.=Sin. 90	595.	Sin. 36°52′	357.=answ.
G :	C ::	G :	C

2. To find an *angle*.

Examp. 1. In the *oblique* triangle ABC, (plate 1. fig. 2.) there are given the *sides* Ac 429, AB 452, and the *angle* B 65°14′, to find the *angle* C.

Thus,

429.	Sin. 65°14′	451.5	Sin. 72°53′=ans.
C :	G ::	C :	G

Examp.

Examp. 2. In the same triangle let the *sides* AB, BC, and the *angle* C, be given to find the *angle* A.

451.5	Sin. 72°53′		315.	Sin. 41°49 = ans.		
C	:	G	::	C	:	G

Examp. 3. In the *rectangled* triangle BCD, (plate 2. fig. 3.) let the *sides* BC, CD, and the *angle* D, be given, to find the *angle* B.

476.	Sin. 53°08′		357.	Sin. 36.52′ = answ.		
C	:	G	::	C	:	G

Examp. 4. Given in the same *triangle*, the *sides* BD, and BC, to find the *angle* D.

		Rad.				
595.	Sin. 90°		476.	Sin. 53°08′ = answ.		
C	:	G	::	C	:	G

The like to be observed in all other triangles.

N. B. The three angles of every *right* lined triangle are equal to two *right* angles or 180 degrees. Hence,

If two angles of any *oblique* angled triangled, or one of the oblique angles of any *rectangled* triangle be known, the other is known also.

N. A. If in any *oblique* angled triangle, one of the angles be *obtuse*, (viz. greater than 90 degrees) substract it from 180 degrees, and proceed with the *remainder*, (or supplement) as with any other angle.

CHAP.

CHAP. VI.

Of the Diſpoſition *of the* Primes *and* Intermediates *on the Lines of* Tangents, K, L, *and* M, *and how to find the* natural *Tangent of any* Arch *or* Angle *thereby.*

SECT. I. *Of the* plane *Tangent* K.

THESE are put on in ſuch manner, as that with the lines B and C, they will compoſe a table of *natural* tangents.

N. B. All tangents leſs than 45 degrees, are called *lower* tangents, and are found in the very ſame manner as the *natural* ſines. Thus:

Place 1000 on C, to the *radius* or tangent 45 on K; then will the lines become a table of *natural* tangents, from the tangent of 10 minutes to 45 degrees, to the *radius* 1000.

Now, if you ſuppoſe 1000 on C to be 10000, then have you Mr. *Sherwin*'s Table of *natural* Tangents, from 10 minutes, up to the *radius*.

Thus:

Againſt the tangent	30° 58′	you have	6000	its natural tangent on C.
	26 34		5000	
	8 32		1500	
	2 30		436.6	
	0 30		87.27	

SECT.

SECT. II. *Of the Slides* L *and* M *or Tangents.*

Theſe, as hath been obſerved, are put on the back ſide of the ſlides *ſines* H and I; and, when taken together, are a line of *tangents* like unto K; but continued down to the tangent of 1 *minute* of a degree.

Hence, the natural *tangent* of any arch or angle leſs than 10 *minutes*, will be found by the line A, in the ſame manner as any *ſine* leſs than 10 *minutes* is found thereby. *See chap.* III. *ſect.* 3.

Hence obſerve, all *proportions* between *numbers* and *lower* tangents, will be wrought by the lines K, B and C, in the very ſame manner as proportions between *numbers* and *ſines* are by the lines G, B and C.

SECT. III. *Of the* Diſpoſition *of the upper Tangents on* K, *and how to find the* natural *Tangent of any arch or angle* greater *than* 45 *Degrees.*

Place 100 on C to the *radius* or tangent 45 on K. Now you are to imagine the line of *tangents* K, to be continued to the *right* hand from 45, having thereon the *tangents* 50, 60, 70 degrees, &c. up to 84 degrees, ſtanding againſt the *lower* edge of C, and the reſt of the upper *tangents*, viz. from 84 degrees to 89°20′, ſtanding againſt the ſuppoſed radius next *above* C, &c.

Now, if the line K was thus continued and numbered, the *tangent* of each degree and minute thereon,

thereon, would be juſt ſo far *above* 45 on K, as its complement is *below* it.

That is, the *tangent* 50 degrees, is ſuppoſed to be juſt the ſame diſtance *above* 45, as its *complement* 40 degrees, is *below* 45; and the tangent of 60 degrees is ſuppoſed to be juſt ſo far *above* 45, as its complement 30 degrees is *below* it: alſo the tangent of 70 degrees will be found juſt ſo far *above* the tangent 45, as its complement 20 degrees is *below* it, and ſo of any other tangent.

For this reaſon all the *upper* tangents are numbered *backwards*, from the radius or *tangent* 45 degrees, each degree and minute thereof being placed, or ſuppoſed to be placed, at its *complement* on the line K, as appears by the inſtrument.

Obſerve alſo, that the *intermediates* of each of the primes above 45 degrees, are found at their reſpective *complements* on the line K. Thus,

The *tangent* of 54 degrees is repreſented by its *complement*, viz. the tangent of 36 degrees; the *tangent* of 74 is repreſented by its *complement* 16 degrees: alſo the *tangent* of 87°45′, is found at its *complement* 2 degrees, 15 minutes; and 89 degrees, 20 minutes, by the point 40 minutes on K, and ſo on. Hence,

The line of *upper* tangents is repreſented by the line K, but in an *inverted* order.

Now, if in the preſent poſition of the ſlides, you ſuppoſe 100 on C, to repreſent the radius of a *circle*, and the *natural tangent* of any arch, ſuppoſe of 50 degrees, of the ſaid circle be required;

 it

it is evident from what hath been ſaid, that the ſaid *tangent* muſt be ſuppoſed to be found juſt ſo far *above* 100 on C, as the ſaid *tangent* of 50 degrees is ſuppoſed to be *above* the tangent 45 degrees on K.

But the *tangent* of 50 degrees is ſuppoſed to be juſt ſo far *above* the *tangent* 45 on K, as its *complement* 40 is *below* it. That is,

The *tangent* 45, is juſt ſo far *above* the complement of any *upper* tangent, as ſuch *upper* tangent is ſuppoſed to be *above* the tangent 45.

Hence,

1. To find the *natural* tangent of any arch or angle greater than 45 degrees, and not exceeding 89 degrees, 20 minutes.

RULE.

Place the given *radius* on C, to the given *tangent* on K, then againſt the *radius* or tangent 45 on K, is its *natural* tangent on B or C. Hence the

PROPORTION.

As the *co-tangent* of the required arch on K,
Is to the given *radius* on B or C;
So is the *radius* or tangent 45 on K,
To the *natural* tangent of the ſaid arch on C.

Examp. 1. Let it be required to find the *natural* tangent of 75 degrees, to the radius 100.

Thus:

Tang. 15°	100.	tang. 45°	373.2=anſw.
K :	C ::	K :	C

Note. Tang. 15=co-tangent of 75.

4 *Examp.*

Examp. 2. What is the *natural* tangent of 83 degrees, 30 minutes to the same radius?

The *complement* of 83°30′, is 6°30′,

Therefore

Tang. 6°30′	100.	tang. 45°.	877°6′=anſ.
K :	C ::	K :	C

Examp. 3 What is the *natural* tangent of 87 degrees, 15 minutes, to the radius 100?

Co-tangent of 87°15′=2°45′.

Therefore

Tang. 2°45′	100.	tang. 45°.	2081.=anſ.
K :	C ::	K :	C

Examp. 4. Let it be required to find the *natural* tangent of 89 degrees, 50 minutes, to the radius 10.

Co-tangent of 89°50′=0°10′.

Therefore

Tang. 0°10′ :	10.	tang. 45°. :	3437.=anſ.
K :	B ::	K :	C

The like is to be obſerved of any other tangent.

2. How to find the *natural* tangent of an arch or angle greater than 89 degrees, 50 minutes, to any given radius.

This is to be performed at *two* operations by the following

PROPORTIONS.

Firſt, by the lines A and L, M.

Thus,

As the *co-tangent* of the given arch on L.

Is to the given *radius* on A;

So is the *tangent* of 10 minutes on L,

To a *fourth* number on A.

Secondly, by the lines K and B, C.

Thus,

As the *tangent* of 30 minutes on K,

Is to the above found *fourth* number on B;

So is the *tangent* of 45 degrees on K,

To the *required* tangent on C.

Examp. What is the *natural* tangent of 89 degrees, 56 minutes, to the ſame radius?

Co-tangent of $89°56'.=0°04'$.

Therefore,

	Tang. 04′		10.		tang. 10′.		25.	
I.	L	:	A	::	M	:	A	

Then,

	Tang. 10′		25.		tang. 45°.		8594.=anſw.	
II.	K	:	B	::	K	:	C	

The like to be obſerved of any other *tangent.*

Note. Any other *tangent*, within the compaſs of the line A, may be made uſe of inſtead of the *tangent* of 10 minutes.

CHAP.

CHAP. VII.

Of Proportions by the Lines of Tangents *and* Numbers.

OF these there are three sorts, viz.

1. *Lower* tangents compared with *lower* tangents.
2. *Upper* tangents compared with *upper.*
3. *Upper* and *lower* tangents compared with *each other.*

SECT. I. Lower *Tangents compared with* lower *Tangents.*

I. In proportions of *tangents* to *numbers.*

These are performed in the very same manner as those of *sines* to numbers, viz. by the following

PROPORTION.

As the first given *tangent* on K,
Is to the given *number* on B or C;
So is the other given *tangent* on K,
To the fourth *proportional* on B or C.

Examples.

1. Ascending.

Examp. 1. Given the *tangents* 40 minutes, and

4 degrees, 30 minutes, and the *number* 3.5, to find the fourth *proportional*.

Thus,

Tang. 40′ 3.5 tang. 4°30′ 23.67 = answ.
K : C :: K : B first *above*

Examp. 2. Given the *tangents* 40 minutes, and 9 degrees 45 minutes, and *number* 3.5; what is the fourth *proportional*?

Tang. 40′ 3.5 tang. 9°45′ 51.68 = answ.
K : C :: K : B first *above*

Examp. 3. Let the given *tangents* be 40 minutes, and 36 degrees, 15 minutes, and *number* 3.5; what is the other number or fourth *proportional*?

Tang. 40′ 3.5 tang. 36°15′ 220.5 = answ.
K : C : K : C second *above*

2. Descending.

Examp. 1. Given the *tangent* 40 degrees, and 8 degrees, 15 minutes, and *number* 345, to find the fourth *proportional*,

Thus,

Tang. 40° 345. tang. 8°15′ 23.35 = answ.
K : C :: K : C first *below*

Examp. 2. Let it be required to find the fourth *proportional number* to the *tangents* 40 degrees, and 30 minutes, and *number* 345.

Tang. 40° 345. 30′ 3.588 = answer.
K : C :: K : C second *below*

Examp.

Examp. 3. Suppose it be required to find the fourth *proportional* to the *tangent* 40 degrees, and 5 minutes, and *number* 345.

N. B. The second *tangent* being less than 10 minutes, the line A must be made use of, and consequently, the answer will be found by *two* operations, as hath been taught above. *See preceding chapter.*

Thus,

Tang. 40°	345.	10′	1.196 = 4. *number.*	
I. K	: C	:: K	: B	

Then by the lines A and L.

	Tang. 10′	1.196	tang. 5′	.598 = answ.
II.	L	: A	:: L	: A

2. In proportion of *numbers* to tangents.

1. Ascending.

Examp. 1. Given the *numbers* 4.35 and 36.5, and the *tangent* 55 minutes, to find the other *tangent* or fourth proportional.

Thus,

4.35	tang. 55′	36.5	tang. 7° 39′
B	: K	:: B	: K

Examp. 2. Given the *numbers* 4.35 and 258, and *tangent* 55 minutes, to find the other *proportional.*

4.35	tang. 55′	258.	tang. 43° 30′
B	: K	:: C	: K

2. Descending.

Examp. 1. Let the given *numbers* be 25 and .43 and the *tangent* 35°45′; what is the fourth *proportional?*

25.	tang. 35°45′	.43	tang. .43′
C :	K ::	C :	K

Examp. 2. Let it be required to find the fourth *proportional* to the *numbers* 150 and .25, and *tangent* 35 degrees.

Note. This requires *two operations*, thus:

	150.	tang. 35°	.623	tang. 10′
I.	C :	K ::	B :	K

Then,

	.623	tang. 10′	.25	04′ answer
II.	A :	L ::	A :	L.

SECT. II. Upper *Tangents compared with* upper *Tangents.*

1. In proportions of *tangents* to *numbers.*

From what hath been said, nay, from a bare inspection of the lines, it appears, that the fourth *proportional* in this case, will be found by the Rule of Three *Inverse*, by the following

PROPORTION.

As the *second* given tangent on K,
Is to the given *number* on B or C;
So is the *first* given tangent on K,
To the fourth *proportional* on B or C.

Examples.

Examples.

1. Ascending.

Examp. 1. Given the *tangents* 50 degrees, and 75 degrees, and the *numbers* 15, to find the fourth *proportional.*

Thus,

Tang. 75°	15.	tang. 50°	46.97=answ.	
K	: C	:: K	: C	*on*

Examp. 2. Given the *tangents* 50 degrees, and 83 degrees, 30 minutes, and *number* 15, to find the fourth *proportional.*

Tang. 83°30′	15.	tang. 50°	110.4=answ.
K	: B	:: K	: C first *above*

Examp. 3. Let the given *tangents* be 50 de-degrees, and 87 degrees 30 minutes, and the *number* 15; what is the other *proportional* number?

Tang. 87°30′	15.	tang. 50°	288.2=answ.
K	: C	:: K	: C first *above*

Examp. 4. Given the *tangents* 50 degrees, and 89 degreees, 25 minutes, and *number* 4.5, to find the fourth *proportional.*

Tang. 89°25′	4.5	tang. 50°	370.8=answ.
K	: B	:: K	: C 2d *above*

2. De-

2. Descending.

Examp. 1. Given the *tangents* 89 degrees, and 50 degrees, and *number* 375, to find the fourth *proportional.*

Tang. 50°	375.	tang. 89°	7.8 = answer
K :	C ::	K :	C second *below*

Examp. 2. Given the *tangents* 82 degrees, 45 minutes, and 50 degrees, and *number* 475; what is the fourth *proportional?*

Tang. 50°	475.	tang. 82°45′	72. = answer
K :	C ::	K :	C first *below*

2. Of *numbers* to tangents.

PROPORTION.

As the *second* given *number* on B or C,
Is to the given *tangent* on K;
So is the *first* given *number* on B or C,
To the fourth *proportional* on K.

Examples.

1. Ascending.

Examp. 1. Given the *numbers* 7.5 and 24.5, and the *tangent* 50 degrees, to find the fourth *proportional.*

Thus,

24.5	tang. 50°	7.5	tang. 75°35′ = answer
C :	K ::	C :	K

Examp.

Examp. 2. Let the given *numbers* be 1.5 and 25, and the *tangent* 50 degrees; what is the fourth *proportional?*

25.	tang. 50°		1.5		tang. 87° 07′ = answer
C	:	K	::	C	:	K

2. Descending.

Examp. 1. Given the *numbers* 35 and 4.5, and *tangent* 89 degrees, 20 minutes, to find the fourth *proportional.*

4.5		tang. 89° 20′		35.		tang. 84° 50′
C	:	K	::	B	:	K

Examp. 2. Let the given *numbers* be 256 and 4.5, and the *tangent* 89 degrees, 20 minutes; what is the fourth *proportional?*

4.5		tang. 89° 20′		256.		tang. 56° 30′ = ans.
C	:	K	::	C	:	K

SECT. III. Upper and lower *tangents compared with each other.*

Note. When one of the two tangents concerned in any proposition be *greater*, and the other *less* than the *radius* or tangent 45 degrees, each *tangent* must be compared with the *tangent* 45 degrees, and consequently, the *fourth* proportional will be found by *two* operations.

Thus:

Suppose it be required to find the fourth *proportional* to the *tangents* 70 degrees, and 50 degrees, and the *number* 8,

1. Place

1. Place 30 degrees on K to 8 on C, then against 45 degrees on K, is 13.85 on C.

Now, becauſe the tangent 50 degrees is ſuppoſed to be juſt ſo far *above* the *radius* or tangent 45 degrees, as it really is *below* it: therefore,

2. Place 13.85 on C, to *tangent* 50 degrees on K, then againſt *tangent* 45 degrees on is 16.51, the fourth *proportional* on C.

That is,

1. As the *leſſer* tangent is to the given *number*, ſo is tangent 45 degrees to a *fourth* number. And

2. As the *tangent* 45 is to the ſaid *fourth* number; ſo is the *greater* given tangent to the fourth *proportional*.

Which laſt *proportion* will, by the inſtrument, run *inverſely*. Thus,

As the *ſecond* or *greater* tangent, is to the above found *fourth* number; ſo is the *tangent* 45 degrees to the fourth *proportional*.

But any *greater* tangent (ſuppoſe 50 degrees) is to the *tangent* 45 degrees directly, as the *tangent* 45 degrees is to the *complement* of ſuch *greater* tangent, (viz. 40 degrees.)

Hence,

1. The fourth *proportional* to any two given *tangents*, and a given *number*, will be found by the following

PROPORTIONS.

1. Aſcending.

I. As the *leſſer* given tangent on K,
Is to the given *number* on B or C;

So

So is the *tangent* 45 degrees on K,
To a fourth *number* on B or C.

Then,

II. As the *complement* of *greater* given tangent on K,
Is to the said fourth *number* on B or C;
So is the *tangent* 45 degrees on K,
To the fourth *proportional* on B or C.

Examp. 1. Given the *tangents* 20 degrees, and 60 degrees, and the *number* 7.5, to find the fourth *proportional*.

Thus,

	Tang. 20°		7.5	tang. 45°		20.6	
I.	K	:	C	:: K	:	C	

Then,

	Tang. 30°		20.6	tang. 45°		35.7=ans.	
II.	K	:	C	:: K	:	C	

Examp. 2. Given the *tangents* 20 degrees, and 83 degrees, 30 minutes, and *number* 7.5, to find the fourth *proportional*.

	Tang. 20°		7.5	tang. 45°		20.6
I.	K	:	C	:: K	:	C

Then

	Tang. 6°30′		20.6	tang. 45°		180.8=ans.
II.	K	:	B	:: K	:	C

Examp. 3. Let it be required to find the fourth *proportional* to the *tangent* 20 degrees, and 89 degrees, 30 minutes, and *number* .75.

	Tang. 20°		.75	tang. 45°		2.06
I.	K	:	C	:: K	:	C

Then

	Tang. 30′		2.06	tang. 45°		236.=answ.
II.	K	:	B	:: K	:	C

2. De-

2. Descending.

PROPORTIONS.

I. As the *tangent* of 45 degrees on K,
Is to the given *number* on C;
So is the *complement* of the *greater* tangent on K,
To a *fourth* number on B or C.

Then,

II. As the *tangent* 45 degrees on K,
Is to the above found *fourth* number on B or C;
So is the *lesser* tangent on K,
To the fourth *proportional* on B or C.

Note. These are the *converse* of the above proportions, as will evidently appear from the following

Examples.

Examp. 1. Given the *tangents* 60 degrees, and 20 degrees, and *number* 35.7, to find the fourth *proportional*.

	Tang. 45°	35.7	tang. 30°	20.6
I.	K :	C ::	K :	C

Then,

	Tang. 45°	20.6	tang. 20°	7.5=answ.
II.	K :	C ::	K :	C

Examp. 2. Given the *tangents* 83 degrees, 30 minutes, and 20 degrees, and *number* 180.8, to find the fourth *proportional*.

	Tang. 45°	180.8	tang. 6°30′	20.6
I.	K :	C ::	K :	B

Then,

Then,

Tang. 45° 20.6 tang. 20° 7.5=anſw.
II. K : C :: K : C

Examp. 3. Let the given *tangents* be 87 degrees, 30 minutes, and 20 degrees, and *number* 472; what is the fourth *proportional?*

Tang. 45° 472. tang. 2° 30′ 20.6
I. K : C :: K : C

Then,

Tang. 45° 20.6 tang. 20° 7.5
II. K : C :: K : C

Examp. 4. Suppoſe the given *tangents* are 82 degrees, 45 minutes, and 0 degrees, 35 minutes, and *number* 650; what is the fourth *proportional?*

Tang. 45° 650. tang. 7° 15′ 82.7
I. K : C :: K : C

Then,

Tang. 45° 82.7 tang. 35′ .842=anſw.
II. K : C :: K : C 2d *below*

N. B. Compare the *three firſt* of theſe examples with the examples *aſcending*.

Hence, obſerve the *diſtance* of any tangent from its *complement*, is equal to *twice* its diſtance from the *tangent* 45 degrees.

2. In

2. In proportions of *numbers* to tangents:

Or,

How to find a fourth *proportional* to two given *numbers*, and a given *tangent*.

1. Aſcending.

PROPORTION.

As the *firſt* given number on B or C,
Is to the given *tangent* on K;
So is the *ſecond* on B or C,
To the fourth *proportional* on K.

But becauſe the *ſecond* number will always fall *above* the tangent 45 degrees, (*ſee chap.* II. *ſect.* 3.) therefore the *anſwer* muſt be found by the two following

PROPORTIONS.

I. As the given *tangent* on K,
Is to the *leſſer* given number on B or C;
So is the *tangent* 45 degrees on K
To a fourth *number* on B or C.

Then,

II. As the *ſecond* or *greater* given number on C,
Is to the *tangent* 45 degrees on K;
So is the above *fourth* number on B or C,
To the fourth *proportional* on K.

Examp. 1. Given the *numbers* 8.9 and 35, and *tangent* 25 degrees, to find the fourth *proportional*.

	Tang. 25°		8.9		tang. 45°		19.08
I.	K	:	C	::	K	:	C

Then,

Then,

35. tang. 45° 19.08 tang. 61.23 = ans.
II. C : K :: C : K

Examp. 2. Given the *numbers* 2.25 and 126, and the *tangent* 8 degrees, 30 minutes, to find the other *tangent*.

Tang. 8° 30′ 2.25 rad. 15.05
I. K : B :: K : C

Then,

126. rad. 15.05 tang. 83° 11′ = ans.
II. C :: K :: B : K

Examp. 3. Let the *numbers* be .5 and 450, and *tangent* 1 degree; what is the other *tangent*?

Tang. 1° .5 rad. 28.64
I. K : C :: K : C

Then,

450. rad. 28.64 tang. 86.21 = ans.
II. C : K :: B : K

2. Descending.

PROPORTIONS.

I. As the *radius* or tangent 45° on K;
Is to the *first* or greater number on C;
So is the *complement* of the given tangent on K;
To a fourth number.

II. As this *fourth* number on C;
Is to the radius on K;
So is the *other* given number on B or C;
To the fourth *proportional* on K.

Note. These are the *converse* of the two last proportions.

Examp. 1. Let the given *numbers* be 35 and 8.9, and the *tangent* 61.23; what is the fourth *proportional?*

	rad.		35.	*cotan.* 61.23		19.08	
I.	K	:	C	::	K	:	C

Then

	19.08		*rad.*		8.9		*tang.* 25°=answer
II.	C	:	K	::	C	:	K

Examp. 2. Given the *numbers* 126 and 2.25, and *tangent* 83 degrees, 11 minutes, to find the other *tangent*.

	rad.		126	*co-tang.* 83.11		15.05	
I.	K	:	C	::	K	:	B

Then,

	15.05		*rad.*		2.25		*tang.* 8.30=answer.
II.	C	:	K	::	B	:	K

Examp. 3. Suppose the given *numbers* are 450 and .5, and the *tangent* 86 degrees, 21 minutes; what is the fourth *proportional?*

	rad.		450.	*co-tang.* 86°21′		28.7	
I.	K	:	C	::	K	:	B

Then,

	28.7		*rad.*		.5		*tang.* 1°=answer
II.	C	:	K	::	C	:	K

Compare these with the three *preceding* examples.

CHAP.

CHAP. VIII.

Solution of Problems in plane *Trigonometry by the Lines of* Tangents.

Note. THESE lines concern *rectangled* triangles only; and because,

If in a *rectangled* triangle, either of the *legs* or *sides* containing the *right* angle, be made the *radius* of a circle, the other *leg* will be the *tangent* of its *opposite* angle.

Therefore;

All problems relating to *plane* trigonometry, wherein *tangents* are concerned, will be solved by the lines of *tangents* and *numbers*, by the following

PROPORTIONS.

I. As either of the *legs* on C,
Is to the *radius* on K;
So is the other *leg* on B or C,
To the *tangent* of its opposite angle on K;

And,

II. As the *radius* on K,
Is to the *greater* leg on C;
So is the *tangent* of the angle *adjacent* on K,
To the other *leg* on B or C.

Hence,

III. As the *co-tangent* of the *greater* angle on K,
Is to its *adjacent* leg on B or C;

So

So is the *radius* on K,
To the other *leg* on C.

Examples.

1. Two *legs* being given, to find *either* angle?

Given in the *right* angled triangle B, C, D, (plate 1. fig. 3.) the *legs* Bc 476, and CD 357, to find the *angle* B.

Thus,

476. *rad.* 357. *tang.* 36° 52′ answer
C : K :: C : K

Note. If the *greater* angle D be required, the proportion will run thus, viz. As, Dc, the *adjacent* leg is to the *radius*; so is CB, to the tangent of its *opposite* angle D.

Which by the instrument will run inverted

Thus,

476. *rad.* 357. *co-tang.* 36° 52′ = 53° 08
C : K :: C : K

2. The *greater* leg and its *adjacent* angle being given, to find the other *leg*.

Given in the same *triangle* the leg BC 476, and its *adjacent* angle B 36 degrees, 52 minutes, to find the *leg* CD.

Thus,

rad. 476. *tang.* 36° 52′ 357 = answ.
K : C :: K : C

3. The

3. The *lesser* leg and its *adjacent* angle being given, to find the other *leg*.

Given the *leg* CD 357, and *angle* D 53 degrees, 08 minutes, to find the *leg* BC.

Thus,

tang. 36.52 357. *rad.* 476 answer
K : C :: K : C

N. B. The like is to be observed in every other *right angled* plane triangle.

THE END OF THE FIFTH PART.

A

KEY

TO THE

MODERN SLIDING RULE.

PART VI.

Containing the Use of the sliding Sines *and* Tangents *in* spheric *Trigonometry.*

INTRODUCTION.

Of spheric *Trigonometry, with the* Use *of the Lines of* Sines *and* Tangents *in finding the Parts of a* spherical *Triangle.*

SECT. I. *Of* spheric *Trigonometry:*

SPHERIC *trigonometry* treateth of the proportion of the *sides* and *angles* of sperical *triangles.* It is chiefly applied to the use of finding the *distances* and *situation* of places or points, on the globe of the *earth*; or in the sphere of the *heavens.*

Now, on the *terrestial* globe, also in the *celestial* sphere, there are an infinite number of *imaginary*

 circles,

circles, which are usually divided into *two* sorts, viz. *greater* and *lesser*.

Great circles of the globe or sphere, are the *equator*, the *ecliptic*, the *horizon*; and all *meridians*, or any other imaginary *circle*, which is supposed to divide the *globe* or *sphere* into *two* equal *parts* or *hemispheres*.

Lesser circles of a sphere or globe, are such as are supposed to divide it into *two unequal parts*; such are all parallels of *latitude* on the *terrestial*, and *almicanters* or parallels of *altitude* on the *celestial* sphere or globe.

Now, the *periphery* of every *great circle* as above, is supposed to be divided into 360 *equal* parts or degrees; each of which degrees is supposed to be *subdivided* into 60 *equal* parts, called *minutes* of a degree; by which *degrees* and *minutes*, the *distance* of any particular *points* of the terrestial and celestial *sphere*, from each other are measured.

SECT. II. *Of the Measure of the* Sides *of a spheric Triangle.*

If on the *convex* surface of the *earth*, or in the *concave* surface of the *heavens*, you imagine *three* points, so situated as not to be in a *right* or straight line, and at the same time you conceive *three great* circles to cut or cross each other in the said *points*; such *circles* will form a *spheric* triangle, the *measure* of either of whose *sides*, will be the number of *degrees* and *parts* contained (thereon) between

between the *two* points, through which such *side* doth pass.

Hence it follows, that the *sides* of every *spheric* triangle drawn in *plano*, do represent parts of the *arcs* of *three great* circles on the globe of the *earth*, or in the sphere of the *heavens*.

Hence also, the *sides* of *spherical* triangles are represented by *tables*, or *lines* of sines and tangents, and not by *numbers* as in *plane* trigonometry.

SECT. III. *Of the* Measure *of a spherical Angle.*

If you imagine two *great* circles to cross each other, in any given *point* of the surface of the celestial or terrestrial *sphere*, and continued, they will cross each other also exactly at the *opposite* point of the said *sphere*, and form 4 angles at each of the said *points*.

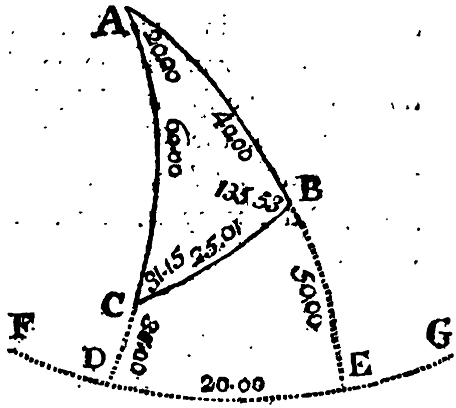
If another *great* circle be supposed to cut or cross the above said *two* circles in the middle of, or at equal distance from the said two *points*, it will cut the former *circles* at *right angles* (and will be) at 90 degrees, from each of the said *points*.

Now, if this last *circle* be supposed to be divided into *degrees* and *minutes*, the *difference* of degrees and minutes cut thereon, by the above two *circles*, is the measure of the *angle* formed by the said two circles at each of the above said *points*.

That is, the *measure* of a *spheric* angle is the *difference* of degrees, cut on a *great* circle by the

two

two *sides* or arcs forming the *angles*, each arc or side being produced to 90 degrees from the *angular* point.



Thus in the *spheric* triangle annexed, whose sides are AB 40 degrees, AC 60 and BC 25 degrees, 01 minute, if you produce AB to E, and AC to D, viz. till the sides AE and AD each become 90 *degrees* from the *angular* point A; and through the said *points* D and E, be drawn a *great* circle FG, the number of *degrees* contained between the points D and E of the said *circle* F, G, will be the *measure* of the *angle* A, viz. of 20 degrees.

Note. The *like* to be observed of either of the other *angles*.

From what hath been said, it follows, that

All problems relating to *spheric* trigonometry, will be solved by the lines of *sines* and *tangents* only, viz. by the lines of *sines* and *sines*, or by the lines of *tangents* and *sines*, in the very same manner as those which relate to *plane* trigonometry, are by the lines of *sines* and *numbers*, or *tangents* and *numbers*, as will be shewn below.

CHAP.

CHAP. I.

Of the Solution of rectangled *spheric Triangles, by* Lord Neper's *Catholic Proposition.*

SECT. I. *Of the* Parts *of a* spherical *Triangle.*

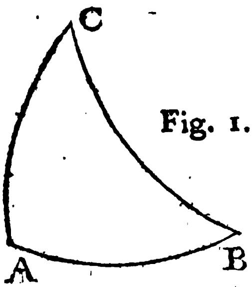


Fig. 1.

1. IN every *rectangled* spherical triangle there are, besides the *right* angle, five things which the Lord *Neper* calls the *five circular parts* of a *spheric triangle*; among which, (the right angle not being reckoned) the two *legs* AB, and AC, are supposed to join together. (See fig. 1.)

2. Any one of these five *circular* parts, may by supposition, be made a *middle* part, and then the two *circular* parts, which are next to that *middle* part, are called *extremes conjunct*; and the other two *circular* parts, remote from that assumed *middle* part, are called *extremes disjunct*.

3. In every case, *two* of the aforesaid five *circular* parts, are always given to find a *third*; and of these three things, (viz. two given and one required) one is the *middle* part, and the other two are either extremes *conjunct* or *disjunct*.

N. B. In the above triangle, and also in the three following, the right angle is at the point A.

SECT.

SECT. II. *To know the* mean *and* extreme *Parts.*

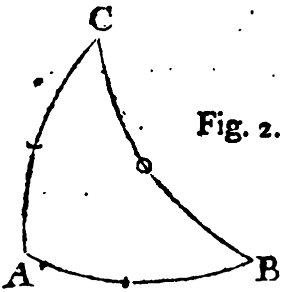


Fig. 2.

1. If *one* of the three *terms* given stands by itself, severed from the other *two* on both sides, as the *side* BC from the *sides* AB and AC, by the interposition of the *angles* B and C; that shall be the *middle* part, and the other two *circular* parts, AB and AC, are the extremes *disjunct*. (*See* fig. 2.)

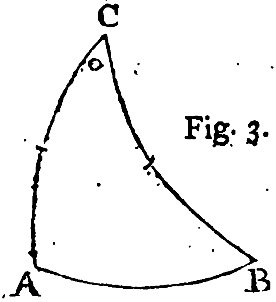


Fig. 3.

2. If the *terms* do immediately adhere together as the *side* BC, *angle* C, and *side* AC, the *middle term* C doth easily shew the *middle* part; and the *extreme* terms BC and AC, are the *extreme* parts *conjunct*. (*See* fig. 3.)

The parts of a *rectangled* spherical triangle being thus distinguished, observe the *Universal Proposition* following by the aforesaid Lord, called

The CATHOLIC PROPOSITION,

Viz. The *sine* of a *middle* part and *radius* are reciprocally *proportional* with the *tangents* of the extremes *conjunct*, and with the *cosines* of the extremes *disjunct*. That is,

First,

First, For extremes *conjunct*.

As the *radius*
Is to the *tangent* of one *extreme*;
So is the *tangent* of the other *extreme*,
To the *sine* of the *middle* part.

Secondly, For extremes *disjunct*.

As the *radius*
Is to the *cosine* of one *extreme*;
So is the *cosine* of the other *extreme*,
To the *sine* of the *middle* part.

Hence,

Note. When the *middle* part is to be found, the *radius* is the *first* term in the proportion.

But if either of the *extremes* is required to be found, then the *other* extreme must be the *first* term therein. That is,

First, For extremes *conjunct*.

As the *tangent* of the given *extreme*
Is to the *radius*;
So is the *sine* of the *middle* part,
To the *tangent* of the required *extreme*.

Secondly, For extremes *disjunct*.

As the *cosine* of the given *extreme*,
Is to the *radius*;
So is the *sine* of the *middle* part,
To the *cosine* of the required *extreme*.

N. B.

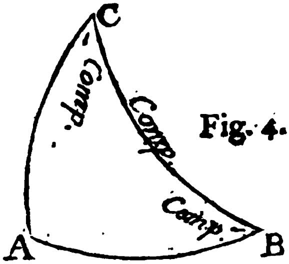


Fig. 4.

N. B. Those three of the *circular* parts which are more *remote* from the *right* angle, as the *angles* B and C, with the *side* AC; (viz. the *hypothenuse* and the two *oblique* angles) *Lord Neper* changeth into their *complements.* (See fig. 4.)

Therefore, if the *middle* part, or either of the *extremes conjunct* be the *hypothenuse*, or either of the *oblique* angles, instead of *sine* and *tangent* in the proportions, use *cosine* and *cotangent.*

N. A. When a *complement* in the *proportion* doth chance to concur with, or fall on a *complement* in the *circular* parts, you must always take the *sine* itself, and *tangent* itself, instead of *co-sine* and *co-tangent* in the *circular* parts: because the *cosine* of the *cosine* is the *sine* itself, and the *cotangent* of the *cotangent* is the *tangent* itself.

Examples.

1. In extremes *conjunct.*

Examp. 1. Given in the *triangle* ABC (fig. 2.) the *perpendicular* AC, and the *hypothenuse* BC; to find the *angle* C.

Because the three *circular* parts given are *conjunct*, and the *middle* part is required to be found, therefore by the *catholic* proposition, the *proportion* will run thus:

rad. : tang. AC :: tang. BC : sin. C

But because the *side* BC, and *angle* C, are both *complements*, (see fig. 4.) therefore the *proportion* will run thus;

As rad. : tang. AC :: cotang. BC : cosine C

Examp. 2. In the *triangle* ABC, (fig. 2.) let the *hypothenuse* BC, and *angle* C be given; to find the *perpendicular* AC.

Here, the three *circular* parts given are *conjunct* also, but an *extreme* AC, is required; therefore by the *catholick* proposition it will be thus:

As tang. BC : rad. :: sin. C : tang. AC.

But because BC and C are both *complements*, (see fig. 4.) therefore it will be

cotang. BC : rad. :: cosin. C : tang. AC

2. In extremes *disjunct*.

Examp. 1. Given the *hypothenuse* BC, and *angle* B, (fig. 3.) to find the *perpendicular* AC.

By the *catholic* proposition, thus,

rad. : cosin. B :: cosin. BC : sin. AC

But the *angle* B, and *hypothenuse* BC, are both *complements*, (*see above*) therefore it will be,

rad. : sin. B :: sin. BC : sin. AC

Examp. 2. Given the *side* AC, and *angle* B, (fig. 3) to find the *hypothenuse* BC.

By the *catholic* proposition, thus,

cosin. B : rad. :: sin. AC : cosin. BC

But because the *side* BC, and also the *angle* B, are *complements*, (see fig. 4.) therefore it will be

sin. B : rad. :: sin. AC : sin. BC

Hence

Hence observe, if the *extremes* be *disjunct*, the proportion will be performed by *sines* only; but if they are *conjunct*, then by *sines* and *tangents*.

CHAP. II.

Solution of the sixteen *Cases of* right *angled* spherical *Triangles by the* Instrument.

(Plate III.)

SECT. I. *Of the* eight *Cases of Extremes* disjunct.

PROPORTIONS.

1. To find the *middle* part.

AS the *radius* or sine of 90 degrees,
Is to the *cosine* of the given extreme;
So is the *cosine* of other extreme,
To the *sine* of the middle part.

2. To find either of the *extremes*.

This is the converse of the former, therefore
As the *cosine* of the given extreme,
Is to the *radius* or sine 90;
So is the *sine* of the *middle* part,
To the *cosine* of the other extreme.

Exampl

Examples.

1. For the *middle* part.

This hath *three* varieties.

Case 1. In the spherical *triangle* ABC, (fig. 1.) let the *base* AC be 27 degrees, 54 minutes, and the *perpendicular* BC 11 degrees, 30 minutes, to find the *hypothenuse* AB?

Here the *middle* part is required, and it falls on a *complement* in the *circular* parts; (*see chap.* I.) therefore the last line in the proportion will run thus:

Viz. 4 to the *cosine* of the middle part, that is,

	BC		AC	
rad. :	cosin. 11.30 ::		cosin. 27.54 :	cosin. AB

By the instrument, thus,

rad.	sin. 78°30′	sin. 62°06′	sin. 60° = cosi. 30°
G :	I ::	G :	I

Answer 30 degrees.

Case 2. Given the *base* AC 27 degrees, 54 minutes, and the *angle* A, at the base, 23 degrees, 30 minutes; to find the angle B at the *perpendicular*. (fig. 2.)

The *extreme* A, and *middle* part B, both fall on *complements*; therefore the third and fourth lines in the *proportion* will run thus:

3. So is the *sine* of the other extreme,

4. To the *cosine* of the middle part.

Thus,

	AC	A	
rad. :	cosin. 27°54′ ::	sin. 23°30′ :	cosin. B

P

By

By the inſtrument,

rad. ſin. 62°06′ ſin. 23°30′ ſin. 20°38′=coſi.69°22′
G : I :: G : I

Anſwer 69 degrees, 22 minutes.

Caſe 3. Let the *hypothenuſe* AB 30 degrees, be given with the *angle* A 23 degrees, 30 minutes, to find the *perpendicular* BC. (fig. 3.)

Here both *extremes* are *complements* in the circular parts; therefore the ſecond and third lines in the *proportion* muſt be read thus:

2. To the *ſine* of one of the extremes;
3. So is the *ſine* of the other extreme.

Thus by the inſtrument,

rad. ſin. 30°00′ ſin. 23°30′ ſin. 11.30
G : I :: G : I

Anſwer 11 degrees, 38 minutes.

2. For an *extreme*.

This hath five *varieties*.

Caſe 1. Given the *baſe* AC 27 degrees, 54 minutes, and the *hypothenuſe* AB 30 degrees, 0 minutes, to find the *perpendicular* BC. (fig. 4.)

Note. Becauſe the *middle* part falls on a *complement* in the *circular* parts; therefore the third *line* in the *proportion* muſt be read thus;

3. So is the *coſine* of the middle part.

That is,

AC AB
Coſ. 27°54′ : rad. :: coſ. 30°00′ : coſ. BC

By

By the inſtrument.

Sin. 62°06′ rad. ſin. 60°00′ coſ. 78°30′=ſin. 11°30′
I : G :: I : G

Anſwer 11 degrees, 30 minutes.

Caſe 2. Given the *perpendicular* BC, 11 degrees 30 minutes, and the *angle* A 23 degrees, 30 minutes, to find the *angle* B. (fig. 5.)

Here the *middle* part, and alſo the required *extreme*, are both on *complements*; therefore the *third* and *fourth* line in the proportion will run thus:

3. So is the *coſine* of the middle part,
4. To the *ſine* of the required extreme.

Therefore,

Coſ. 11°30′ : rad. :: coſ. 23°′30 : ſin. B

By the inſtrument.

Sin. 78°30′ rad. ſin. 66°30′ ſin. 69°22′
I : G :: I : G

Anſwer 69 degrees, 22 minutes.

Caſe 3. Given the *angles* A 23 degrees, 30 minutes, and B 69 degrees, 22 minutes, to find the *baſe* AC. (fig. 6.)

The *extreme* given, and the *middle* part, are both *complements*; therefore the *firſt* and *third* lines will be read thus:

1. As the *ſine* of the given extreme,
3. So is *coſine* of the middle part.

That is,

Sin. 23°30′ : rad. :: coſ. 69°22′ : coſ. AC

 By

By the inſtrument.

Sin. 23°30′ rad. ſin. 20°38′ coſ. 62°06′=27°54′

I : G :: I : G

Anſwer 27 degrees, 54 minutes.

Caſe 4. Given the *perpendicular* BC 11 degrees, 30 minutes, and the *angle* A 23 degrees, 30 minutes, to find the *hypothenuſe*. (fig. 7.)

Both the given *extremes* being *complements*, the *firſt* and *fourth* lines in the proportion will be

1. As the *ſine* of the given extreme,
4. To the *ſine* of the required extreme.

Thus by the inſtrument.

Sin. 23°30′ rad. ſin. 11°30′ ſin. 30°00′

I : G :: I : G

Anſwer 30 degrees, 00 minutes.

Caſe 5. Given the *baſe* AC 27 degrees, 54 minutes, and the *hypothenuſe* AB 30 degrees, to find the *angle* B. (fig. 8.)

Here the two given *extremes* alſo fall on *complements* in the circular parts; therefore the *firſt* and *laſt* line in the *catholic propoſition* will run thus:

1. As the *ſine* of the given extreme,
4. To *ſine* of the required extreme.

Thus,

Sin. 30°00′ rad. ſin. 27°54′ ſin. 69°22′

I : G :: I : G

Anſwer 69 degrees, 22 minutes.

Theſe are all the *varieties* which can happen in extremes *disjunct*.

N. B.

N. B. From the laſt *caſe*, and alſo *caſe* 3. *page* 210. you may obſerve, that the *ſides* of every ſpheric triangle are in *direct* proportion to each other, as are the *ſines* of their *oppoſite* angles.

SECT. II. *Of the* eight *Caſes of extremes* Conjunct.

Note. Theſe cannot be ſolved on the inſtrument by the *catholic* propoſition, as it above ſtands, (though they may by the tables of *ſines* and *tangents*,) becauſe three *tangents* are required thereby to be taken, and *one ſine*.

I ſhall therefore here ſhew, how the ſaid *propoſition* may be ſo tranſpoſed, as to become applicable to the *inſtrument*.

This is effected by *inverting* the two firſt lines of the *proportion*, and taking the *complement* of the firſt *tangent* inſtead of the *tangent* itſelf; for

As the *tangent* of any arch is to the *radius*,
So is the *radius* to the *cotangent* of the ſame arch*.

Hence,

If the *extremes* are *conjunct*, the ſolution of problems by the *inſtrument* will be performed on the *lines* of *ſines* and *tangents*, by the following

PROPORTIONS.

1. To find either of the *extremes*.

As the *radius* or ſine of 90 degrees,
Is to the *cotangent* of the given *extreme*;

* 13 Euc. 6 and 31. Euc. 3.

So is the *sine* of the *middle part*,
To the *tangent* of the required *extreme*.

2. To find the *middle* part.

This the *converse* of the former, therefore

As the *cotangent* of one of the *extremes*,
Is to the *radius*, or sine 90;
So is the *tangent* of the other *extreme*,
To the *sine* of the *middle part*.

N. B. The same rule is to be observed with regard to *complements* in the *circular* parts as in the above *proportions* by sines.

1. To find an *extreme*.

This hath *five* varieties.

Case 1. Given the *base* AC 27 degrees, 54 minutes, and the *angle* A at the base 23 degrees, 30 minutes, to find the *perpendicular*. (fig. 9.)

Here is only one *complement* in the *circular* parts, viz. the given *extreme*; therefore the *second* line in the *proportion* must be read thus:

2. To the *tangent* of the given *extreme*,

Thus by the instrument,

Rad.	tang. 23°30′	sin. 27°54′	tang. 11°30′
G :	M ::	G :	M

Answer 11 degrees, 30 minutes.

Case

Case 2. Given the *hypothenuse* AB 30 degrees, and the *angle* at the base 23 degrees, 30 minutes, to find the *base*. (fig. 10.)

The given *extreme* and the *middle* part are both *complements*; therefore the *second* and *third* lines will run thus:

2. To the *tangent* of the given extreme;
3. So is the *cosine* of the middle part.

That is,

Rad. : tang. 30°00 :: cos. 23°30′ : tang. AC

By the instrument,

Rad.	tang. 30°00′	sin. 66°30′	tang. 27°54′
G :	M ::	G :	M

Answer 27 degrees, 54 minutes.

Case 3. Given the *hypothenuse* AB 30 degrees, and the *angle* A at the base 23 degrees, 30 minutes, to find the *angle* B. (fig. 11.)

Here all the terms fall on *complements* in the *circular* parts; therefore the *three* last lines will run thus:

2. To *tangent* of the given *extreme*;
3. So is the *cosine* of the *middle part*,
4. To the *cotangent* of the required *extreme*.

That is,

Rad. : tang. 23°30′ :: cos. 30°00′ : cotang. B

By the instrument,

Rad.	tang. 23°30′	sin. 60°00′	cos. 20°38′ = 69°22′
G :	M ::	G :	M

Answer 69 degrees, 22 minutes.

Case 4. Given the *base* AC 27 degrees, 54 minutes, and the *perpendicular* BC 11 degrees, 30 minutes, to find the *angle* A. (fig. 12.)

The required *extreme* falls on a *complement*; therefore the *fourth* line will run thus:

4. To the *cotangent* of the required *extreme*.

That is,

Rad. : Cotang. $11^\circ 30'$ :: sin. $27^\circ 54'$: Cotang. A

By the instrument.

Rad. : Tang. $78^\circ 30'$ :: sin. $27^\circ 54'$: Cotang. A

viz. Thus,

Sin. $27^\circ 54'$	tang. $78^\circ 30'$	rad.	Cotang. $66^\circ 30'$
G :	M ::	G :	M

Answer 23 degrees, 30 minutes.

See part V. *chap.* VI. *sect.* 3. *and* VII. *sect.* 2. *and* 3.

Case 5. Given the *base* AC $27^\circ 54'$, and the *angle* A $23^\circ 30'$, to find the *hypothenuse*. (fig. 13.)

Here the *middle part*, and the required *extreme* fall both on *complements*; therefore the *third* and *fourth* line will run thus:

3. So is the *cosine* of the *middle part*,

4. To the *cotangent* of the required *extreme*.

That is,

Rad. : cotang. $27^\circ 54'$ :: cos. $23^\circ 30'$: cotang. AB

By the instrument.

Thus,

Sin. $66^\circ 30'$	tang. $62^\circ 06'$	rad.	cotang. $60^\circ 00'$
G :	M ::	G :	M

Answer 30 degrees. *See as above.*

2. To

2. To find the *middle part.*

This hath *three* varieties.

Case 1. Given the *perpendicular* BC 11°30′, and the *angle* A at the *base* 23°30′, to find the *base*. (fig. 14.)

Here, only one of the *extremes* falls on a *complement* in the *circular parts*; therefore, if that be made the *first* term in the *proportion*, the *first* line will run thus:

1. As the *tangent* of the given *extreme*.

Thus,

Tang. 23°30	rad.	tang. 11°30′	sin. 27°54′
M :	G	:: M	: G

Answer 27 degrees, 54 minutes.

Case 2. Given the *base* AC 27°54′, and the *hypothenuse* 30 degrees, to find the *angle* A at the base. (fig. 15.)

Here one of the *extremes*, also the *middle part* falls on a *complement*; therefore the *first* and *fourth* line will be.

1. As the *tangent* of the given *extreme*,
2. To the *cosine* of the *middle part*.

That is,

Tang. 30°00′	rad.	tang. 27°54′	cosin. 66°30′
M :	G	:: M	: G

Answer 23 degrees, 30 minutes.

Case

Case 3. Given the *angle* B 69°22′, and the *angle* A 23°30′, to find the *hypothenuse*. (fig. 16.)

Here all the *terms* fall on *complements* in the *circular parts*; therefore the *first*, *third* and *fourth* lines in the *proportion* must be read thus:

1. As the *tangent* of one of the *extremes*,
3. So is the *cotangent* of the other *extreme*.
4. To the *cosine* of the *middle part*.

That is,

Tang. 69°22′ : rad. :: cotang. 23°30′ : cosin. AB

Or rather thus,

Tang. 23°30′ : rad. :: cotang. 69°22′ : cosin. AB.

Thus by the instrument,

Tang. 23°30′	rad.	tang. 20°38	cosin. 60°00′
M	: G ::	M	: G

Answer 30.00 degrees.

These are all the *varieties* which can happen in extremes *conjunct*.

Observe,

1. If only one of the *extremes* falls on a *complement* in *circular parts*, make that the *first* term in the proportion.

2. If *both* extremes fall on *complements*, make the *least* of them the *first* term, so will any problem be solved by the *lower* tangents.

Note. The *like* is to be observed in all other *rectangled* spheric triangles.

CHAP.

CHAP. III.

Of oblique *angled* ſpherical *Triangles.*

THE ſolution of problems wherein *oblique* angled *ſpheric* triangles are concerned, depends on the *ſolution* of the foregoing *problems*: for if from any *angle* of an *oblique* angled triangle, an *arch* be let fall (or ſuppoſed to be ſo) perpendicularly on the *oppoſite* ſide, ſuch *arch* will divide the ſaid *triangle* into two *right* angled *triangles.* And,

If this perpendicular *arch* be let fall from the end of a *known* ſide, and ſo as to be *oppoſed* to a known *angle*, viz. in ſuch manner as that *two* of the three given or known parts of the *triangle* may be on one and the ſame *ſide* of the ſaid *perpendicular*; then may the *parts* of the ſaid *oblique* triangle be found at *two* operations, by ſome or other of the above *proportions*; but I ſhall here ſubjoin ſome *theorems* and *corollaries*, whereby all the caſes of *ſpheric* triangles may be ſolved.

Theorem I. In every *right angled* ſpherical triangle, the proportion is, as *radius* is to *ſine* of the *hypothenuſe*, ſo is the *ſine* of the angle at the baſe, to the *ſine* of the *perpendicular*: and, as the *radius* is to the *tangent* of the hypothenuſe, ſo is *coſine* of the *angle* at the baſe, to the *tangent* of the baſe.

Corol. 1. Hence it follows, that the *ſines* of the angles of every *oblique* angled triangle, are to each

each other directly as the *sines* of their opposite sides.

Corol. 2. It follows also, that in any *two right* angled spheric triangles having one leg *common*, the *tangents* of the hypothenuses are to each other *inversely*, as the *cosines* of the *adjacent* angles.

Theorem II. In any right angled *spherical* triangle it will be, as *radius* to *cosine* of one leg, so is the *cosine* of the other leg to *cosine* of the *hypothenuse*.

Corol. Hence, if *two* right angled *spherical* triangles have the same *perpendicular*, the cosines of their *hypothenuses* will be to each other directly, as the *cosines* of their *bases*.

Theorem III. In any right angled *spheric* triangle it will be, as the *radius* is to the *sine* of either *oblique* angle, so is the *cosine* of the *adjacent* leg, to the *cosine* of the *opposite* angle.

Corol. Hence, in *right* angled *spheric* triangles, having the same *perpendicular*, the *cosines* of the angles at the *base* will be to each other *directly*, as the *sines* of the *vertical* angles.

Theorem IV. In any *right* angled *spheric* triangle it will be, as the *radius* is to the *sine* of the base, so is the *tangent* of the *angle* at the base, to the *tangent* of the perpendicular.

Corol. Hence it follows, that in *right* angled *spherical* triangles, having the same *perpendicular*, the *sines* of the bases will be to each other *inversely*, as the *tangents* of the angles at the bases.

Theorem

Theorem V. In any right angled *spheric* triangle it will be, as the *radius* is to the *cosine* of the *hypothenuse*, so is the *tangent* of either *oblique* angle, to the *cotangent* of the other: and, as the *radius* is to the *cosine* of either of the *oblique* angles, so is the *tangent* of the hypothenuse, to the *tangent* of its adjacent perpendicular.

Lemma. As the *sum* of the *sines* of two *unequal* arcs is to their *difference*, so is the *tangent* of *half* the sum of those *arcs*, to the tangent of *half* their *difference.*

And, as the *sum* of their *cosines* is to their *difference*, so is the *cotangent* of *half* the same arcs, to the *tangent* of *half* the *difference* of the same arcs.

Theorem VI. In any *spherical* triangle it will be, as the *cotangent* of half the *sum* of the two sides is to half their *difference*, so is the *cotangent* of half the *base* to the *tangent* of the distance of the perpendicular from the middle of the base.

Corol. Since the last proportion, by permutation, becomes tangent $\frac{AB+BE}{2}$: cotangent AD :: *tangent* $\frac{AB-BE}{2}$: tangent CD, and since, as the *tangent* of any two *arcs* are *inversely* as their *cotangents*, it follows, that *tangent* AD : *tangent* $\frac{AB+BE}{2}$ :: tangent $\frac{AB-BE}{2}$: tangent CD. That is, the *tangent* of half the *base* is to the *tangent* of half the *sum* of the sides, as the *tangent* of half the *difference* of the sides, is to the *tangent* of the distance of the *perpendicular* point from the middle of the *base.*

Theorem

Theorem VII. In any *spheric* triangle it will be, as the *cotangent* of half the sum of the *angles*, at the *base* is to the *tangent* of half their *difference*, so is the *tangent* of half the *vertical* angle to the *tangent* of the *angle* which the *perpendicular* makes with the line *bisecting* the *vertical* angle.

N. B. Any *three* parts of any *oblique* angled *spherical* triangle (except the three *angles*) being given, the rest may be known by the above *theorems* and *corollaries*.

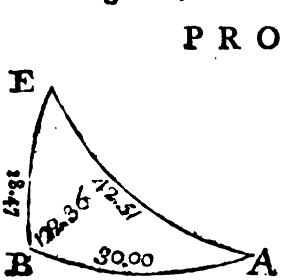
See Barrow's Universal Dictionary of Arts and Sciences.

CHAP. IV.

Of the Use *of the Line of* versed Sines.

THE *use* of this line is too extensive to be treated of in this place: I shall therefore give my reader an example thereof, in the *solution* of one *problem* whose great use in *astronomy* and *navigation*, is alone sufficient to recommend it.

PROBLEM.



Given the three *sides* of an *oblique* angled *spherical* triangle, ABE, viz. AB 30 degrees, BE 18 degrees, 47 minutes, and AE 42 degrees, 51 minutes, to find either of the *angles*, for instance, the *angle* E.

I. RULE.

RULE.

Add all the *sides* together, and from half their *sum* subtract the *side* opposed to the required *angle*, and observe the *difference*: then by the lines of *sines*.

Thus,

As the *sine* of half the *sum* of the sides,
Is to *one* of the *sides* containing the *angle*;
So is the *sine* of the *other* containing side,
To a *fourth* sine.

Then by the *versed* sines.

Thus,

Place the above found *fourth* sine on I to the beginning or *point* O of the line of *versed* sines, viz. right against the *sine* 90 degrees of the line G, then against the above found *fourth* sine on H, is the *versed* sine of the *angle* sought.

Thus,

The sides *containing* the required *angle*, are - - -	AB=30°00′ BE=18 47
The side *opposed* to the required *angle*, is - - -	AE=42 51
The *sum* of all the sides, - - -	=91 38
Half ditto, - - - - -	=45 49

Therefore,

Sin. 45°49′	sin. 30°00′	sin. 18°47′	sin. 12°58′.
G :	I ::	G :	I

Then as

Then 2.

Half the *sum* of all the sides *lessened* by the side AE, - - - $\Big\} = 2^\circ 58'$

Therefore,

Sin. $12^\circ 58'$		O	fin. $2^\circ 58'$			$122^\circ 36'$
I	:	V.S.	::	H	:	V.S.

Answer 122 degrees, 36 minutes.

THE END OF THE SIXTH PART.

APPENDIX.

APPENDIX.

Description and Use of four different Instruments whereon the inverted *Lines are peculiarly and respectively adapted to the Use of* Timber-merchants, Carpenters *and* Sawyers, Shipwrights, Bricklayers *and* Glaziers, *in measuring of Superficies and Solids at* one Operation; *whereby the Time and Trouble of rectifying the Instrument is saved.*

N. B. EACH of these *instruments* being no other than the *inverted* line of the former, properly *rectified* and fixed to each respective *particular* use; it will be sufficient here to give a *description* of each *instrument*, and refer the reader to the *examples* of the use of its corresponding *divisors*, exhibited *part* II. *chap.* IX.

DESCRIPTION.

I. Of the *Timber-merchants*, *Carpenters* and *Sawyers* Instrument.

1. On one of the *broader* planes of this instrument, is put the *single* line D, in a *doubled* manner as usual.

2. On the *opposite* plane to this, and above the *slide*, is put an *inverted* line marked Bst: and is the line for measuring of a *stock* of boards.

3. On the *narrower* plane, next under this, is an *inverted* line marked □Tim.·., for measuring of *rectangled* superficies and prisms, or *square timber*.

N. B. This line is also marked K:·; for *solids* of *sawyers* works.

4. On the *opposite* edge to this is put an *inverted* line marked ⊖Tim.·: and is the line for measuring of *circular* and *elliptical* superficies, and *prisms* of timber, &c. having such bases.

5. On the *lower* edge, viz. under the slide on each plane except D, is put the *radius* A. *See as above, sect.* 1.

II. Of the *Shipwrights* Instrument.

1. On one of the *broader* planes, and abutting against the *upper* edge of the slide, is put an *inverted* line marked Shw.·. for gauging of *ships* of *war*.

2. On the *opposite* plane is put an *inverted* line, marked Shm: for gauging of *merchant* men.

3. On one of the *narrower* planes is put an *inverted* line mark Shst: for *statute* gauging of ships. *See sect.* 3.

4. On the *lower* edge of each of the above *planes*, is put the radius A.

5. On the other *plane* of this instrument is put the *line* D.

6. On the *backsides* of the *slides* B and C, is put the *line* E.

III. Of

III. Of the *Bricklayers* Instrument.

1. On one of the *broader* planes of this instrument, and above the *slide*, is put an *inverted* line marked BW.·. and is the line for finding the content of any *walling*, in rods or poles, the dimensions being taken in *feet* and *decimal* parts.

2. On the *opposite* plane to the abovesaid, is put an *inverted* line marked Bn°: for finding the number of *statute* bricks, required to build a wall of any number of bricks *thickness*, whose *height* and *length* is given in *feet* and decimal parts. *See sect.* 2.

3. On the *lower* edge of each of the abovesaid planes, is put the *radius* A.

N. B. On the other two *planes* may be put any *lines* or tables at pleasure.

IV. Of the *Glazier's* Instrument.

1. On one of the *broader* planes hereof, and abutting against the *upper* edge of the slide, is put an *inverted* line marked G: which is the line for *lights*, whose *height* is taken in *feet* and decimal parts, and *breadth* in *inches* and decimal parts.

2. On the *opposite* plane to this is put an *inverted* line marked G.·: for *lights* whose *height* and *breadth* are both taken in *inches* and decimal parts.

3. On one of the *narrower* planes is an *inverted* line marked G. for lights, whose *height* and

and *breadth* are both taken in *feet* and decimal parts. *See sect.* 3.

4. On the *lower* edge of each of the abovesaid *planes*, is put the *line* A.

N. B. On the other *plane* may be put the line D, or any other useful *line* or *tables*.

FINIS.

Plate

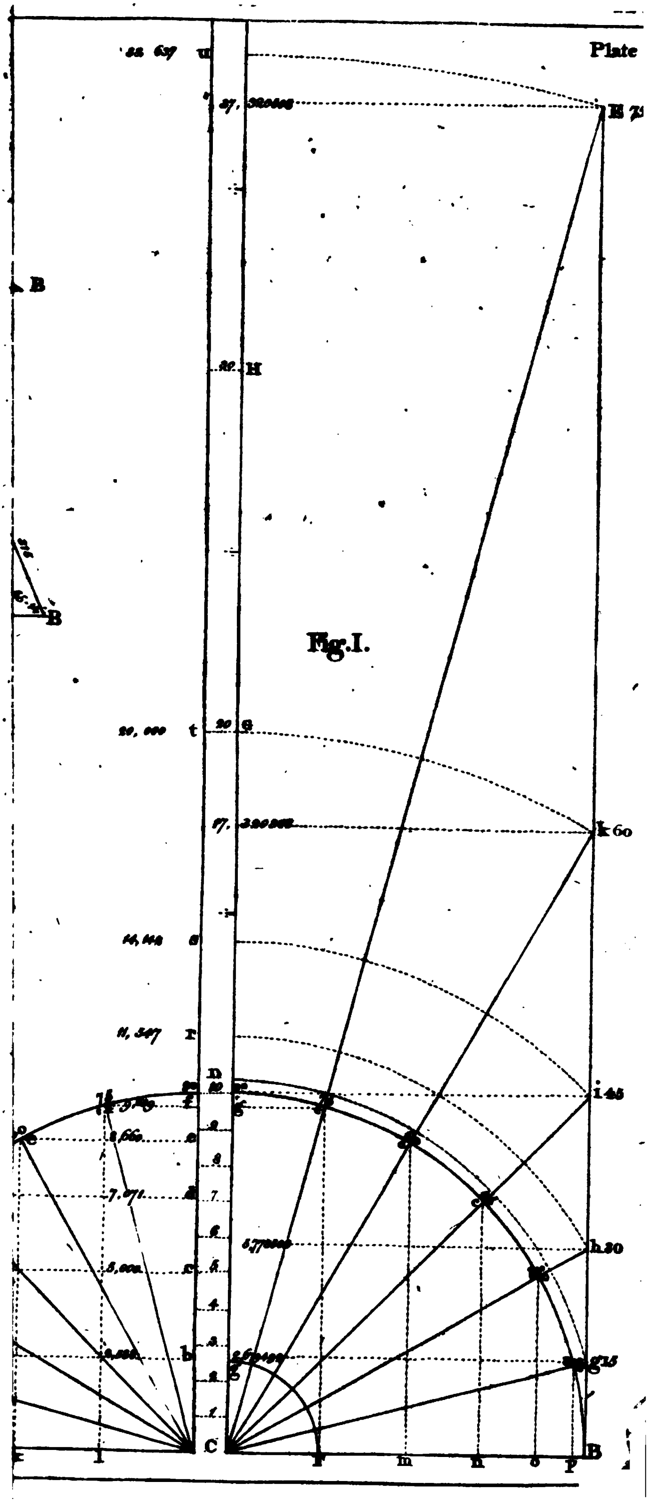
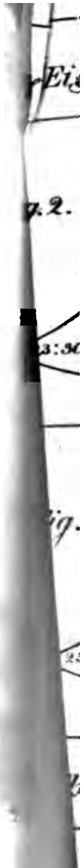
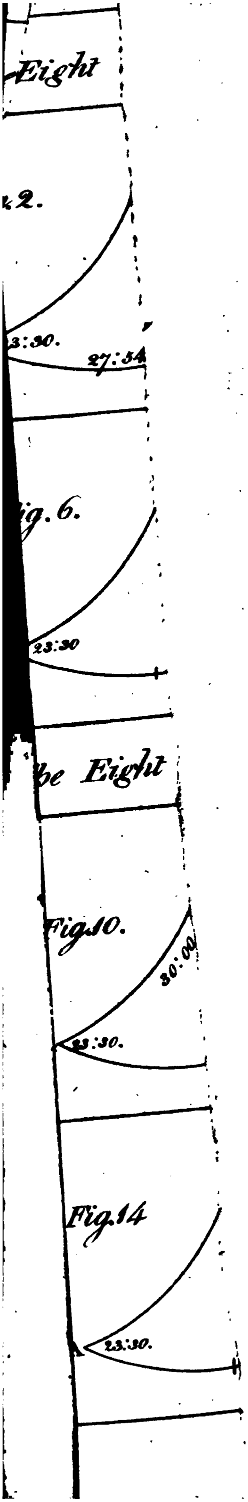


Fig. I.

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2.
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Fig. 10.
30:00
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Fig. 14
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